DOCUMENT RESUME

ED 297 993 SE 049 626

AUTHOR Sanok, Gloria; Vissa, Jeanne
TITLE Adventures in Mathematics: The N-dimensional Journeys. Monograph Number 3.
INSTITUTION Association of Mathematics Teachers of New Jersey, Vineland.
PUB DATE 87
NOTE 60p.; For Monograph 1, see ED 270 321. For Monograph 2, see ED 262 989.
AVAILABLE FROM AMTNJ Monograph, c/o Gloria Sanok, Wayne Public Schools, Wayne, NJ 07470 ($5.00).
PUB TYPE Guides - Classroom Use - Guides (For Teachers) (052) -- Guides - Classroom Use - Materials (For Learner) (051)

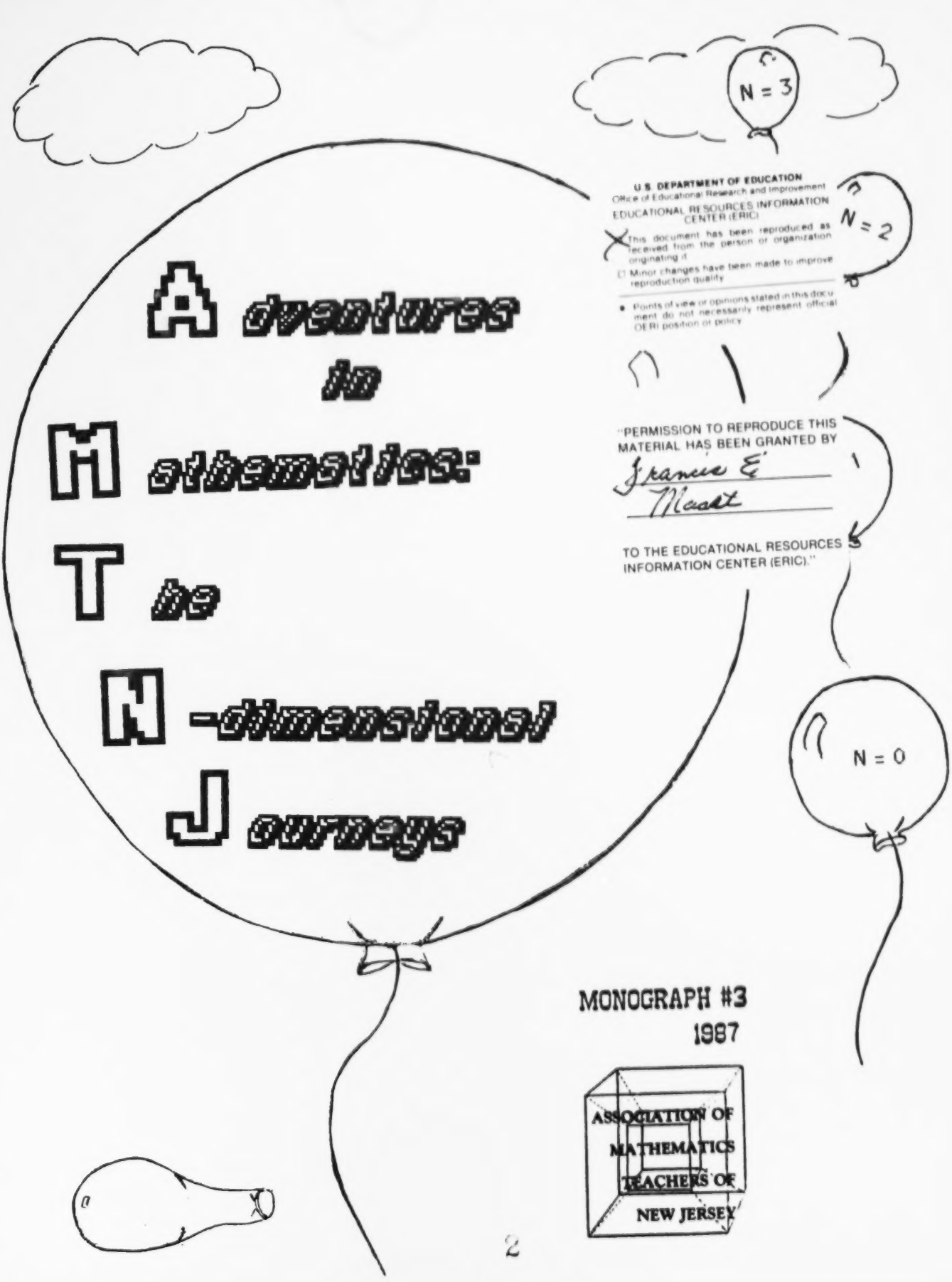
EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS Elementary Education; *Elementary School Mathematics; *Geometric Concepts; Instructional Materials; *Learning Activities; Lesson Plans; *Mathematical Enrichment; Teaching Guides; Worksheets
IDENTIFIERS *New Jersey

ABSTRACT

This monograph presents geometric activities designed to foster curiosity and inquiry on the part of both students and teachers as they explore geometric ideas for 0 to 3 dimensions. Activities can be used in whole class presentation, small groups, or individual study. Grade levels span K-8. Suggested questions promote communication and sharing, and answers are included. The guide sheets include objectives to focus the activity, needed materials, historical background, directions, calculator usage where appropriate, and extensions of the topics in the activity. Lessons may be duplicated for classroom use. The activities concern: "From 0-D to 1-D: What's the Point?" (introduction to point and line); "The Geometry of Distortions" (topological transformations); "Curve Designs" (optical illusions); "Coordinate Distortions" (coordinate geometry); "String Along" (ratio); "Going Around in Circles" (pi); "Golden Ratio" (ratio and proportion); and "The Shadow from the Nile" (indirect measurement). (MNS)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED297993



Adventures in Mathematics: To the N-dimensional Journeys

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

X This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY

Frances E Mendt

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

MONOGRAPH #3
1987

ASSOCIATION OF MATHEMATICS TEACHERS OF NEW JERSEY

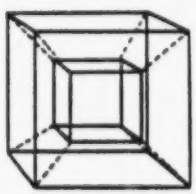
SE 049 626

Adventures in Mathematics: The N-dimensional Journeys

by

Gloria Sanok, Ph. D. and Jeanne Vissa, Ed. D.
Authors and Editors

Carolyn Canfield
Contributing Editor



©1987 by the Association of Mathematics Teachers of New Jersey

Additional copies of this Monograph are available at $5.00 per copy
(made payable to AMTNJ) and can be obtained by writing to:

AMTNJ Monograph
c/o Gloria Sanok
Wayne Public Schools
Wayne, NJ 07470

INTRODUCTION

It is with pleasure that AMTNJ presents our third monograph. As you go on this geometric journey into the world of dimensions from zero to three, you and your students will experience the fun of working together. The activities will enable you to develop the complex thinking skills of inquiry, reasoning, and analyzing solutions with your students.

The artistic shaping of geometry in this monograph would not have been possible if it were not for the creativity, initiative, enthusiasm, and hard work over the last two years of the co-authors and editors, Dr. Gloria Sanok of the Wayne Public Schools and Dr. Jeanne Vissa of the White Plains Public Schools. The mathematics teachers of New Jersey are grateful and appreciative that you have shared your insight on enriching mathematics learning with us.

On behalf of AMTNJ, I would like to thank the contributing editor, Carolyn Canfield of the West Milford Township Schools for her invaluable suggestions, writings, and faithful professional help.

I would like to give special thanks to Diane Eriksen, Dr. Janet Caldwell, Dr. Anthony Piccolino and Dr. Edward Kenny and Dorothy Kenny for their constructive help in giving frank responses to the manuscripts sent for their critical comments and also, to James Jaekle for his computer advice and expertise.

As you and your students work together on the activities in this monograph, there is an opportunity for you to begin a fascinating voyage into the riches of mathematics.

Ellen Mary Brockmann, Ph. D.
President, AMTNJ
April, 1987

Thoughts on Enriching Mathematics Learning

Enrichment in mathematics stretches the student's sphere of knowledge beyond the standard curriculum. The purpose of this monograph is to foster curiosity and inquiry on the part of both students and teachers as they explore geometry on a journey into a world of zero, one, two, and three dimensions. This exploration will require searching for not only similarities and differences but also generalizations and exceptions. The investigation includes: problem solving, history of mathematics, sets, probability, statistics, tables and graphs, number patterns and properties, ratio and proportion, fraction and decimal equivalents.

Activities can successfully be used in whole class presentation, small groups, or individual study. Suggested questions have been included that will foster inquiry and which will allow children themselves to develop a successful process for solution. In mathematics it is important to provide opportunities for children to communicate their ideas both verbally and symbolically. Children are particularly able to benefit from sharing with a partner or discussion in a small group. It is also important for children to learn to be dependent on one another for ideas, rather than to always rely on a teacher-directed solution. When a frustrating situation occurs, teachers may give an answer--and allow children to think out and test a process that would result in that answer. If the teacher feels a need to re-establish the learner's confidence, he or she can ask the student to construct a similar problem. Both teachers and students must become sensitive to the reality that a student skilled in computation may need compensatory techniques to solve a problem in geometry, or vice versa.

The guide sheets included in this monograph are organized with:
Objectives to focus the activity
Materials needed for the activity
Historical background, where applicable
Directions to facilitate carrying out the activity
Calculator usage suggested, where appropriate
Extensions of the topics in the activity

The lessons of the monograph may be duplicated for classroom use. Answers are also provided at the end of each exercise. Do not include the answer area when duplicating worksheets.

The materials herein may not be used for commercial purpose without written permission from AMTNJ.

This material was prepared in unbound and unstapled form to permit flexible use.

One final comment: give yourself time to become familiar with these activities. Enjoy them with your students. We enjoyed writing them for you.

TABLE OF CONTENTS

Activity Title	Grade	Main Topic	Subtopics	Page
From 0-D to 1-D: What's the Point?	K-8	Introduction to Point and Line	Set Notation Union Intersection	1
The Geometry of Distortions	2-8	Topological Transformations	Open and Closed Curves, Shape Recognition	3
Curve Designs	1-8	Optical Illusions	Shape Recognition Transformations Angles Number Fact Relationships Commutative Property	7
Coordinate Distortions	1-8	Coordinate Geometry	Dilations Multiplications Transformations Rectangular Grids Cylindrical Grids Anamorphic Art	9
String Along	3-8	Ratio	Length Metric Measurement	15
Going Around In Circles	4-8	Pi	Circumference,Diameter Radius Organizing Tables Metric Measurement Determining Averages Probability Statistics History of Mathematics	17
Golden Ratio	5-8	Ratio & Proportion	Shape Recognition Measurement Constructions Number Patterns Fraction/Decimal Equivalence History of Mathematics	23
The Shadow from the Nile	5-8	Indirect Measurement	Similar Right Triangles Ratio and Proportion Measurement History of Mathematics	27

Does It Measure Up?	4-8	Triangle Inequality	Pythagorean Relationship Powers	31
From 1-D to 2-D: Handing It Over	4-8	Perimeter and Area	Total Surface Area Multiplication Ratio Measurement	35
The Problems With Area	4-8	Area	Problem Solving Strategies Congruency Conservation of Area Fraction Parts	37
From 2-D to 3D: All Boxed Up	4-8	Volume	Measurement Formulas Tables and Graphs Inductive Reasoning	39
Shaping Up With Color	4-8	Geometric Patterns	Shape Identification Shape Recognition Fraction Concepts Congruency Probability Similarity Problem Solving Strategies	43
Paper Plate Geometry	3-8	Geometric Shapes	Shape Recognition Transformations Symmetry Tessellations Spatial Perception History of Mathematics Following Directions Listening	47
1-D, 2-D, 3-D: Of Toothpicks and Marshmallows	1-8	Spatial Visualization	Geometric Shapes Analysis (taking apart) Synthesis(putting together) Problem Solving Strategies	51
A Meter Cubed	4-8	Metric Measurement	Estimation Measurement Length Area Volume	57

FROM 0-D TO 1-D: WHAT'S THE POINT?

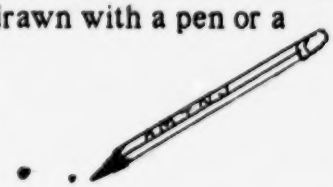
Objective: To identify points and lines

Materials:

straightedge
pencil

Background:

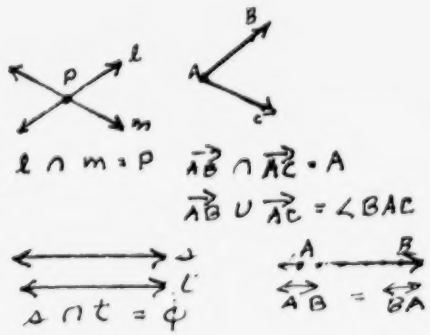
In geometry, a point is a mathematical idea associated with a location. It has zero-dimension. Pictures of points are drawn with a pen or a pencil.



The idea of a point becomes clearer when visualized as the intersection of lines.

A line is thought of as a special set of points. Using this description, a line does not have to be straight. However, a set of points with bends is generally referred to as a curve.

The intersection of two lines and the origin of a ray represent points.



Likewise , the intersection of two rays may be a point or a line segment. Parallel lines have no points in common.

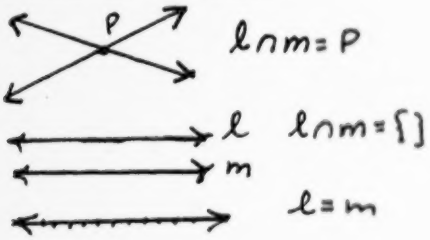
Activity:

1. Name some physical objects that remind you of points.
2. Find all possibilities of the number of points of intersection in a plane for 2 lines; 3 lines; and 4 lines.
3. Draw capital letters of the alphabet made by two distinct line segments.
4. Can you describe the following situations in geometric terms?
   a. a period at the end of a sentence
   b. a bow string
   c. railroad tracks
   d. a violin string and a bow
   e. crossing swords
   f. rungs in a ladder
   g. a bow and arrow
   h. a "point" in time
5. Arrange 7 dots in a plane so that you can draw lines through 6 different sets of dots.

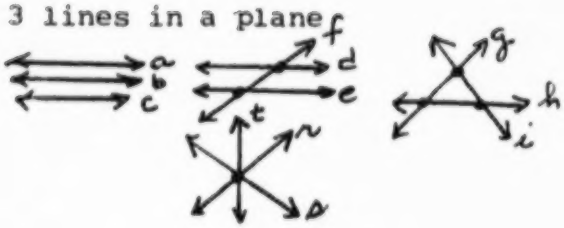
Answers:

1. Answers will vary.

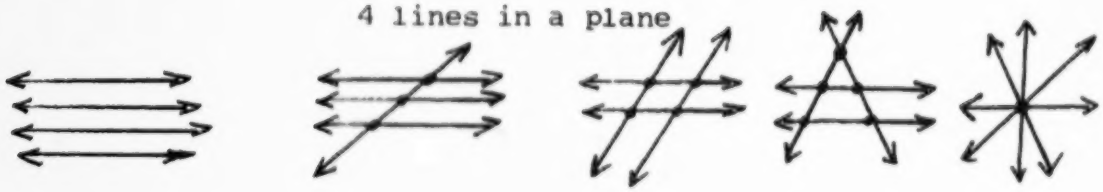
2. 2 lines in a plane



3 lines in a plane



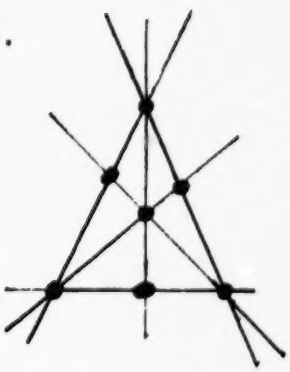
4 lines in a plane



3. L, T, V, X

4. a. a point b. a line c. pair of parallel lines d. a point
e. a point f. parallel lines g. a point

5.



THE GEOMETRY OF DISTORTIONS

Objectives: To recognize the properties of topological transformations

Materials:

two small pieces of paper
a pencil
straightedge
balloons

Background:

Topology is the geometry of distortions. Under a topological transformation, an invariant (no change) is a property about the original figure which is still true with regard to the new figure.

Topological transformations include types of distortions such as twisting, stretching, shrinking, or bending a curve or surface. Tearing or joining are not allowed. For this reason, topology is called rubber sheet geometry.

A simple closed figure like a circle has an inside and an outside. This property will not change even when the figure is distorted (stretched or shrunk).

Another property that does not change is the order of the points along the figure.

Ideas of simple closed curves can be presented quite early to young children:

1. A simple closed curve is topologically equivalent to a circle.
2. Two curves are topologically equivalent if one can be transformed into the other by a topological transformation.
3. If two figures can be twisted or stretched into shape without connecting or disconnecting any points or changing the order, they are said to be topologically equivalent.

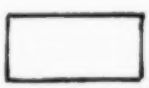
Definitions:

A variant is something that changes.

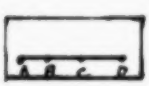
An invariant is something that does not change.

Activity A

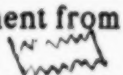
Directions:



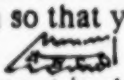
1. Take a small piece of paper.
2. Draw a line segment on it with a pencil and a straightedge.
3. Mark four points on the line segment from left to right.



4. Crumple the paper in your hand.



5. Open enough so that you can see the whole line segment and the marks on it.



The line segment is changed or distorted. What is not changed about it? Is it still connected to itself from one end to the other end ?

10

Activity B

Directions:

1. Draw a circle.
2. Draw a square.
3. Draw a triangle.
   How are the square and the triangle topologically like the circle?
4. Draw a picture of a "blob". Compare it with the others.

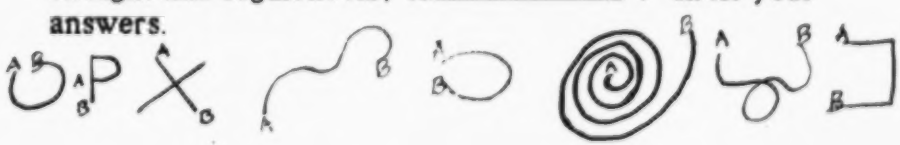
Activity C

Directions:

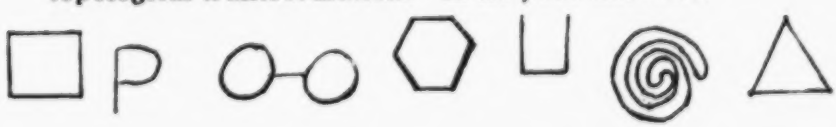
1. Draw a square on the balloon. Mark, in consecutive order, the corners- A,B,C, and D. Stretch it into many kinds of shapes. The rubber balloon can be pulled or twisted about as much as you like as long as you do not tear the rubber or glue two pieces together. The figures continue to separate the surface of the rubber into an inside and an outside. The indicated points keep the same positions in relation to each other along the closed figure. Point A is still between B and D, B is still between A and C, etc.
2. Draw blobs and other shapes such as triangles and circles on the balloon. What kinds of shapes can you stretch them into? In spite of the stretching, what things stay the same?
3. Draw a figure 8. How many insides are there? How many outsides are there? Is this figure topologically equivalent to the square? the triangle? the circle?

Activity D

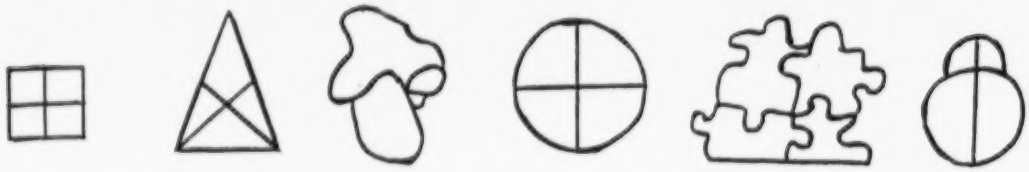
1. Which of the following curves are topologically equivalent to straight line segment AB, A_____B ? Circle your answers.



2. Which of the following can be obtained from a circle by a topological transformation? Circle your answers.



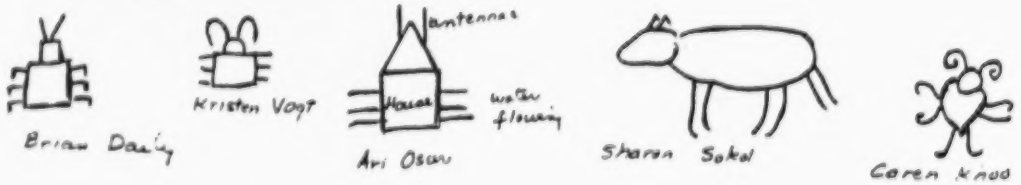
3. List the ways in which the drawings below are alike; different. Look carefully.



4. A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

   a. Which letters are topologically equivalent to the ninth letter in the alphabet?
   b. Which letters are equivalent to the letter O?
   c. The letter E is equivalent to letter G, since E → ⊓ → ᴍ → T → J → G
      What other letters are equivalent to E and G?
   d. Which letters are equivalent to letter K?
   e. Which letters are equivalent to letter Q?

5. Can you turn the beetle into something else? The following drawings were done by students.



Students of Paekanack School, Wayne, NJ

Answers

Activity A

5. Yes, the line is still connected to itself from one end to the other end.

Activity B

3. The square and the triangle are topologically equivalent to the circle because each separates the paper into two regions, the inside and the outside.
4. The blob also has two parts, inside the line and outside the line.

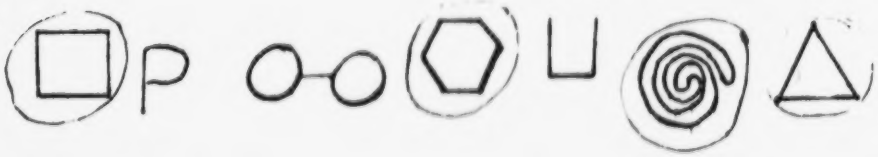
Activity C

2. Answers will vary.
3. two, one, no, no, no.

Activity D

1.

2.



3. The drawings are alike in that they all have four inside parts or regions and one outside.

4. a. C, J, L, M, N, S, U, V, W, Z
   b. D
   c. F, T, Y
   d. X
   e. R
5. Answers will vary.

CURVE DESIGNS

Objective: To use line segments to create the illusion of a curve

Materials: L-shaped diagram (see illustration)
rulers
1 or 2 crayons or markers per child

Activity

Trace the numbered diagram on the back side of the worksheet. Students will work on Side 1 with direction, and on Side 2 independently.

Find the pairs of addends which sum to 10. Connect them with line segments.

The result is an optical illusion of a curve although only straight line segments were used. Make the same optical illusion on Side 2 of the worksheet by connecting the same addend pairs.

Note to the teacher: The intersections of the straight lines form quadrilaterals--four-sided figures-- that may be colored to create a distorted checkerboard effect. If the entire class uses the same two colors, they can then do the transformation geometry activities listed in the Extensions below.

Extensions

Study transformations by having the class complete a square by using Point X of the diagram as the fourth corner. Cut out the square, eliminating the numbers along the edges. Using the collection of students' "tiles", look at the patterns that are produced through translations (repeated slides up, down, left or right), reflections (flips up, down, left and right), and rotations. Students may be reminded of how designs are repeated on wallpaper or gift wrapping paper. You can read more about transformations in Sanok, G. "Living in a World of Transformations", Arithmetic Teacher, April, 1978.

Vary the angle joining the two line segments in the illustration from 90^0 to 60^0 or 120^0. It is possible to create stars, footballs, etc. by combining the "tiles" of the resulting designs. You can also reinforce the properties of figures that are discussed in the Topology section of this monograph.

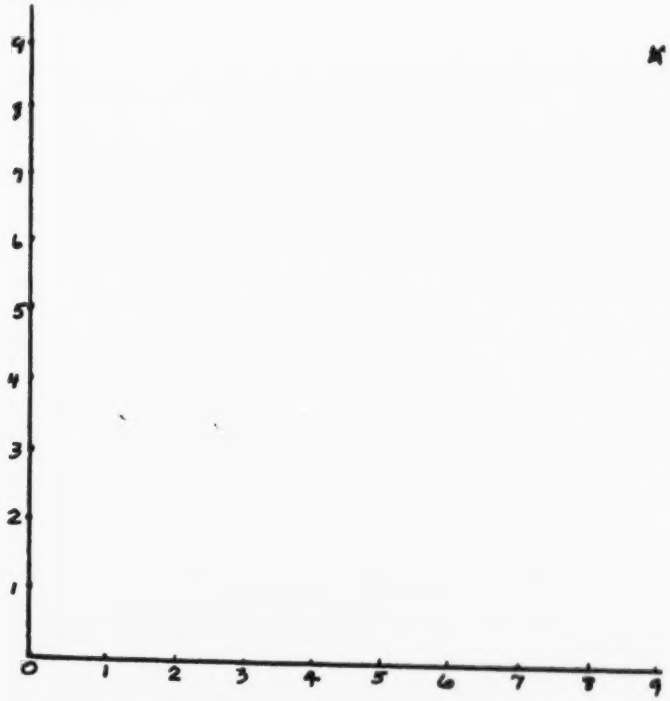
Number fact relationships can be drilled. When a child suggests a pair of addends to connect, the teacher can suggest that the next pair of addends represent the Commutative Property (e.g., after a child suggests 6 + 4 the teacher can lead the class to connect 4 + 6 as well).

Change the numbers along the edges to positive and negative integers. For example, use -4 through +5 along each axis and connect addend pairs that sum to +2.

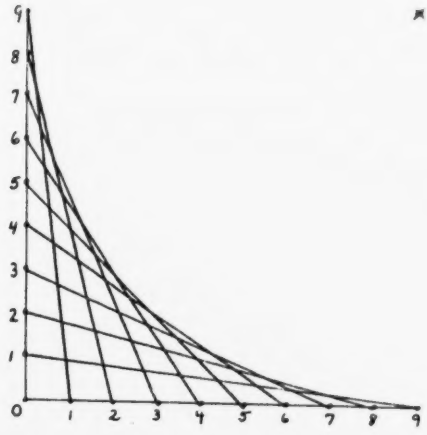
Create thread designs ("Curve Stitching") on a dark background, using punched holes on tagboard or nails on a wooden frame.

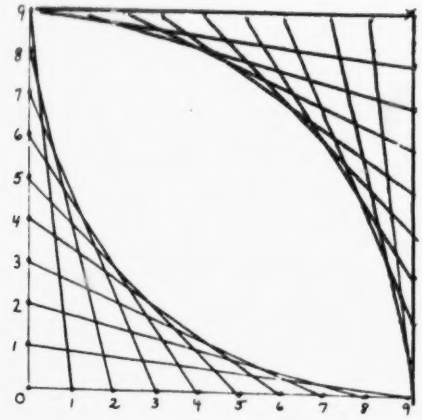
See the Activity, "Shaping Up with Color" to investigate further the coloring of polygon designs.

Illustration



Answers





COORDINATE DISTORTION

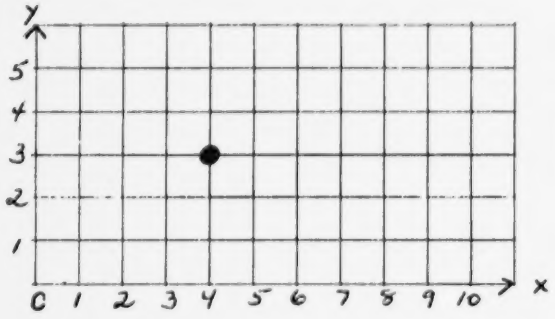
Objective: To examine how altering the basic coordinate system results in figures that are topologically equivalent, but apparent distortions

Materials: 2 copies per student of centimeter square paper, labeled along the bottom edge and left side edge with the numbers 1-20
1 copy per student of rectangular coordinate paper
1 copy per student of cylindrical coordinate paper
Mylar paper cut into squares 20 cm on each edge

Background

Coordinate Geometry was invented by Rene Descartes (renNAY dayCART), a French mathematician of the 17th century. A point's location on a two-dimensional surface is its distance, horizontally and vertically, from a beginning point, called the origin, which is labeled (0,0). In geography, this system was adapted for longitude and latitude on the surface of the earth; in space, three dimensions are needed to locate a point. They are represented by using longitude, latitude, and altitude.

Initially, it must be emphasized to children that a point's coordinates tell its location in relation to (0,0). A number pair tells first the horizontal and then the vertical unit spaces we count in order to locate a point. For convenience and consistency, the horizontal axis has been designated "x" and the vertical axis "y."



(4,3)

A variety of activities can be used to convey to children the structure of the coordinate plane. Among them are shadow projections of a point along the two axes, and giving directions (across and up) to a corner house in a city of one-way streets. Activity sheets using regular square-grid coordinates can be purchased or constructed where alphabet letters are scattered at the intersections of the grid lines, and vocabulary words or rhymes can be discovered by the children as they match letters to given coordinate pairs.

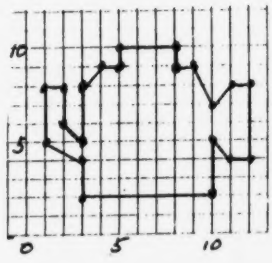
To familiarize themselves with the alternate grids, children can plot the shape formed by the following points connected in order: (1,2) (6,2) (6,7) (1,7). On cm square coordinate paper, the figure is a square, but on other grids the shape is distorted. The results are similar to the shapes obtained by the "balloon geometry" activity suggested as an Extension in The Geometry of Distortions.

Activity A

Draw the following shape on centimeter-square coordinate paper by graphing and then connecting the points corresponding to the number pairs listed below. Work vertically, and cross out each coordinate pair as you use it.

x,y	x,y	x,y
1. (8, 9)	9. (2,8)	17. (12,4)
2. (8,10)	10. (1,8)	18. (12,8)
3. (5,10)	11. (1,5)	19. (11,8)
4. (5, 9)	12. (3,4)	20. (10,7)
5. (4, 9)	13. (3,2)	21. (10,8)
6. (3, 8)	14. (10,2)	22. (9,9)
7. (3, 5)	15. (10,5)	23. (8.9)
8. (2, 6)	16. (11,4)	

Answers

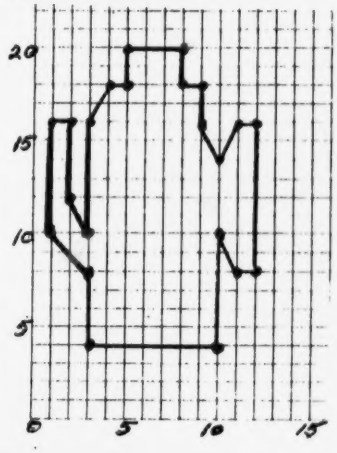


Describe your picture. _____

Activity B

DOUBLE the y-coordinate of each point given in Activity A (i.e., the first point becomes (8,18). Re-draw the design on the second sheet of centimeter-square paper.

x, 2y	x, 2y	x, 2y
1. (8,18)	9. (,)	17. (,)
2. (8,)	10. (,)	18. (,)
3. (5,)	11. (,)	19. (,)
4. (5,)	12. (,)	20. (,)
5. (4,)	13. (,)	21. (,)
6. (3,)	14. (,)	22. (,)
7. (3,)	15. (,)	23. (,)
8. (2,)	16. (,)	

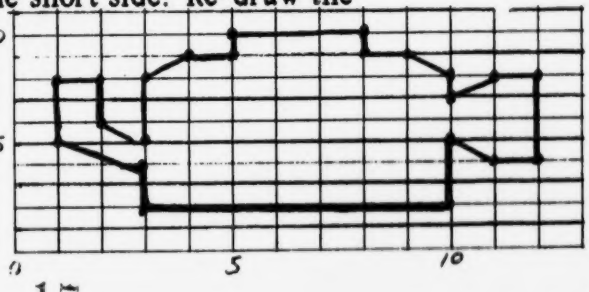


Describe your new picture. _____

What happened to the original design? _____

Activity C

Use a sheet of rectangular coordinate paper with the y-axis marked along the long side, and the x-axis marked along the short side. Re-draw the design, using the coordinates of Activity A.



What happened to the original design? ____

Why did this happen? _____

Activity D

Obtain a sheet of cylindrical graph paper. Note that the x number of the coordinate pair is counted out along the central semi-circle. Use the coordinate pairs given in Activity A, and connect them in order.

Heavily outline your design. Take a piece of Mylar paper that is 20 cm on each edge and roll it into a cylinder and place it to fit the curved x-axis. View the design in the Mylar paper.

What happened to the design? _____

Can you explain why this happened? _____

Extensions

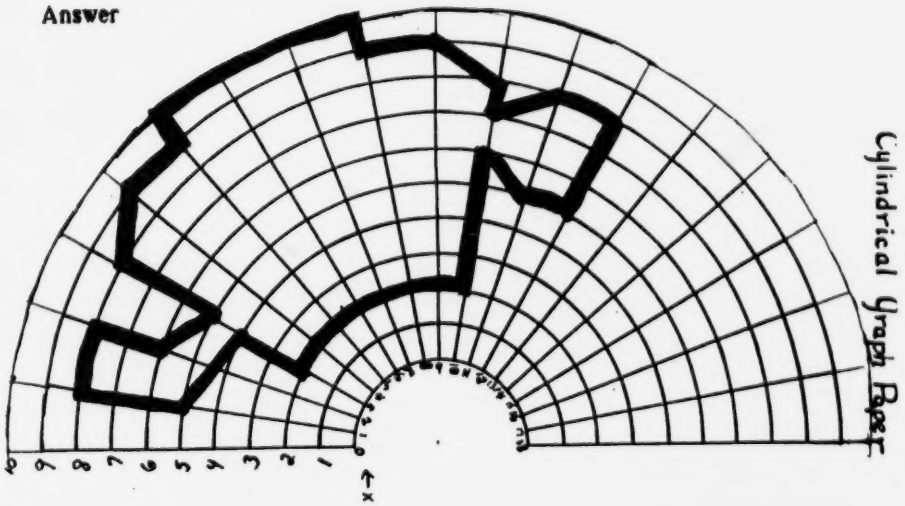
Make designs using coordinate pairs, and then illustrate the designs on the various grid papers used in Activities A-D. Design a creamer and sugar bowl after Activity B.

Introduce the 4-quadrant Cartesian plane with positive and negative coordinates, and adjust the axes on the distorted graph papers accordingly.

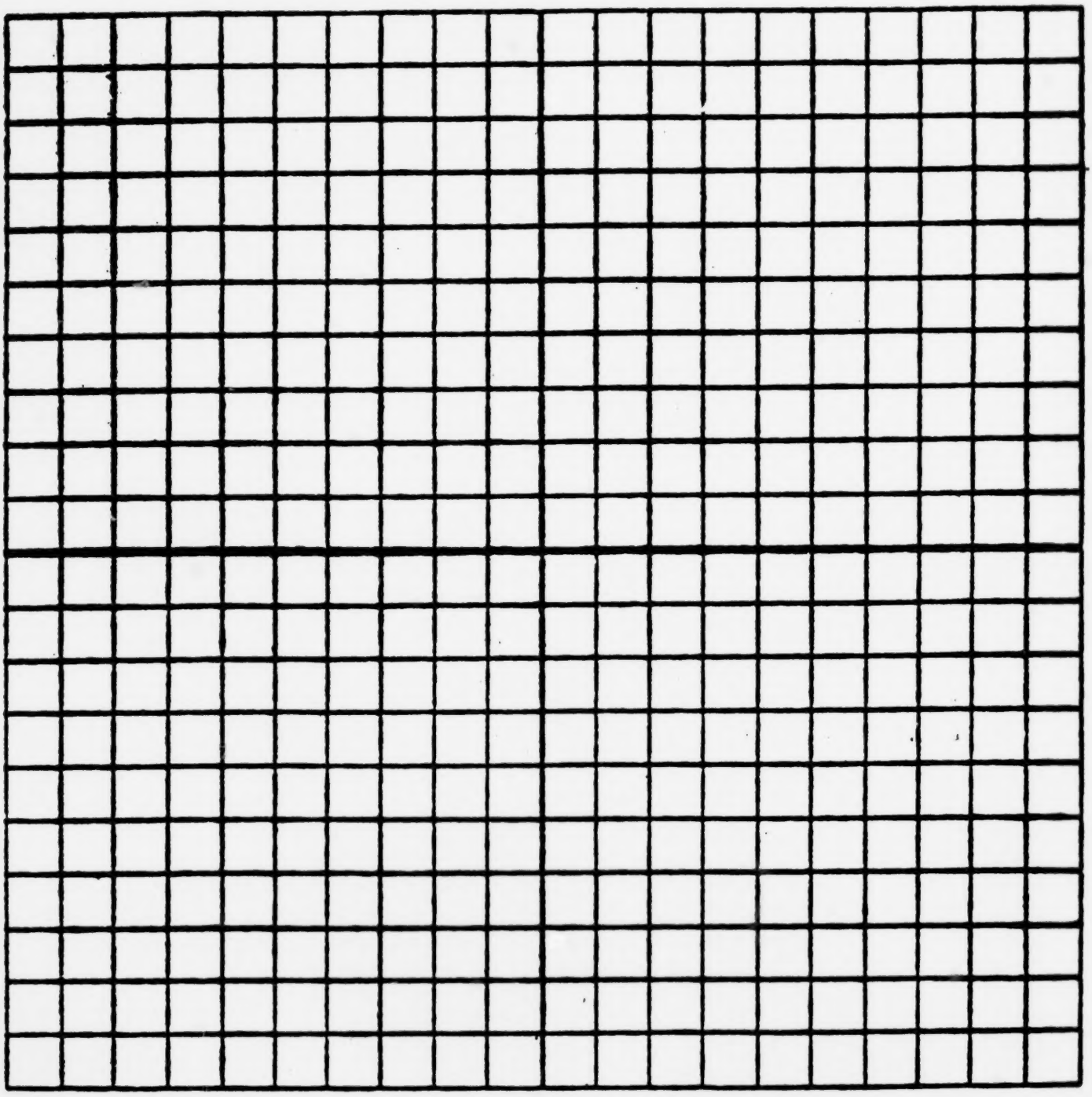
Research "Anamorphic Art," its history, and published artwork.

Ask a professional or amateur artist the grid method that can be used to paint an enlarged canvas from a favorite photograph.

Answer

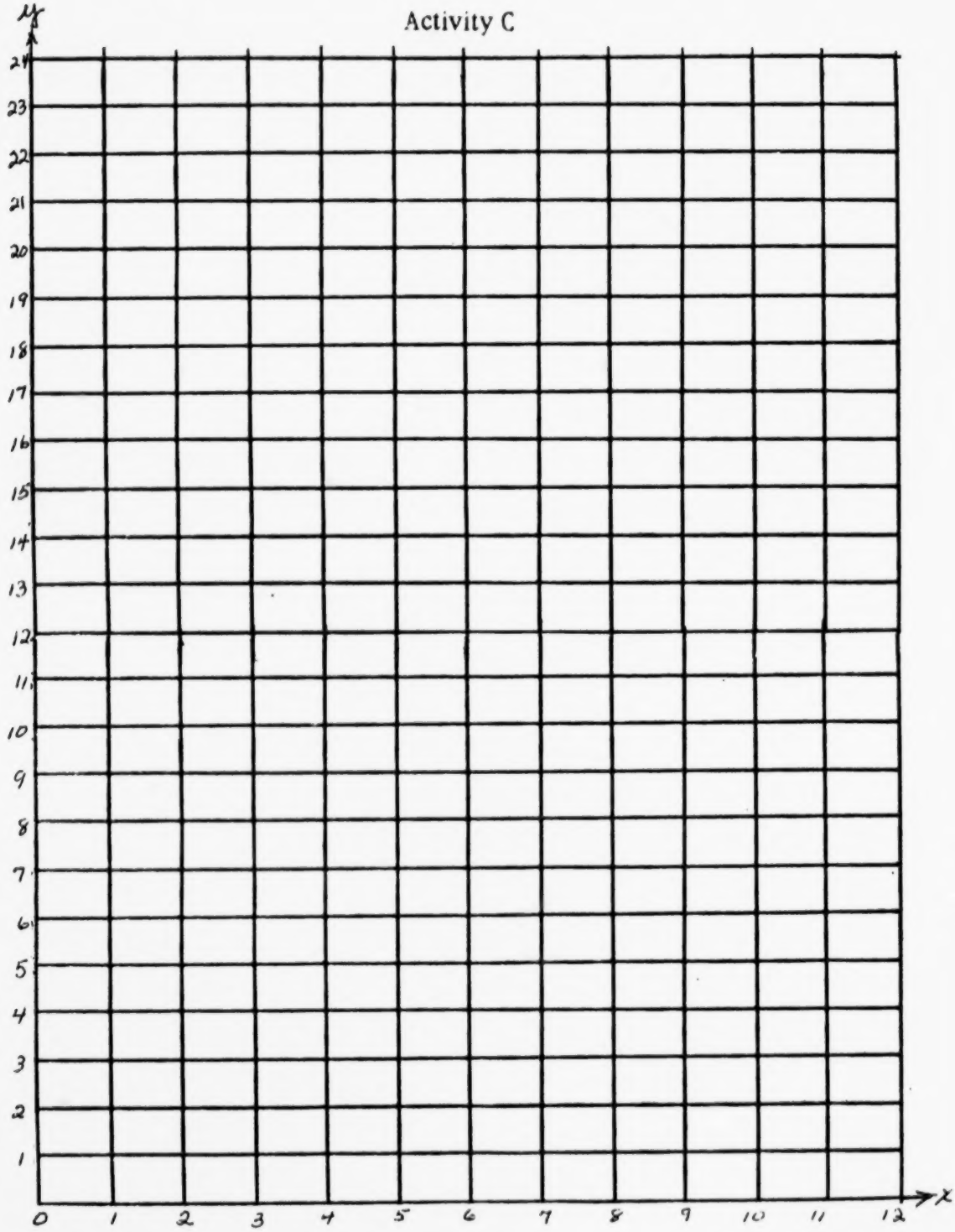


CENTIMETER GRAPH PAPER



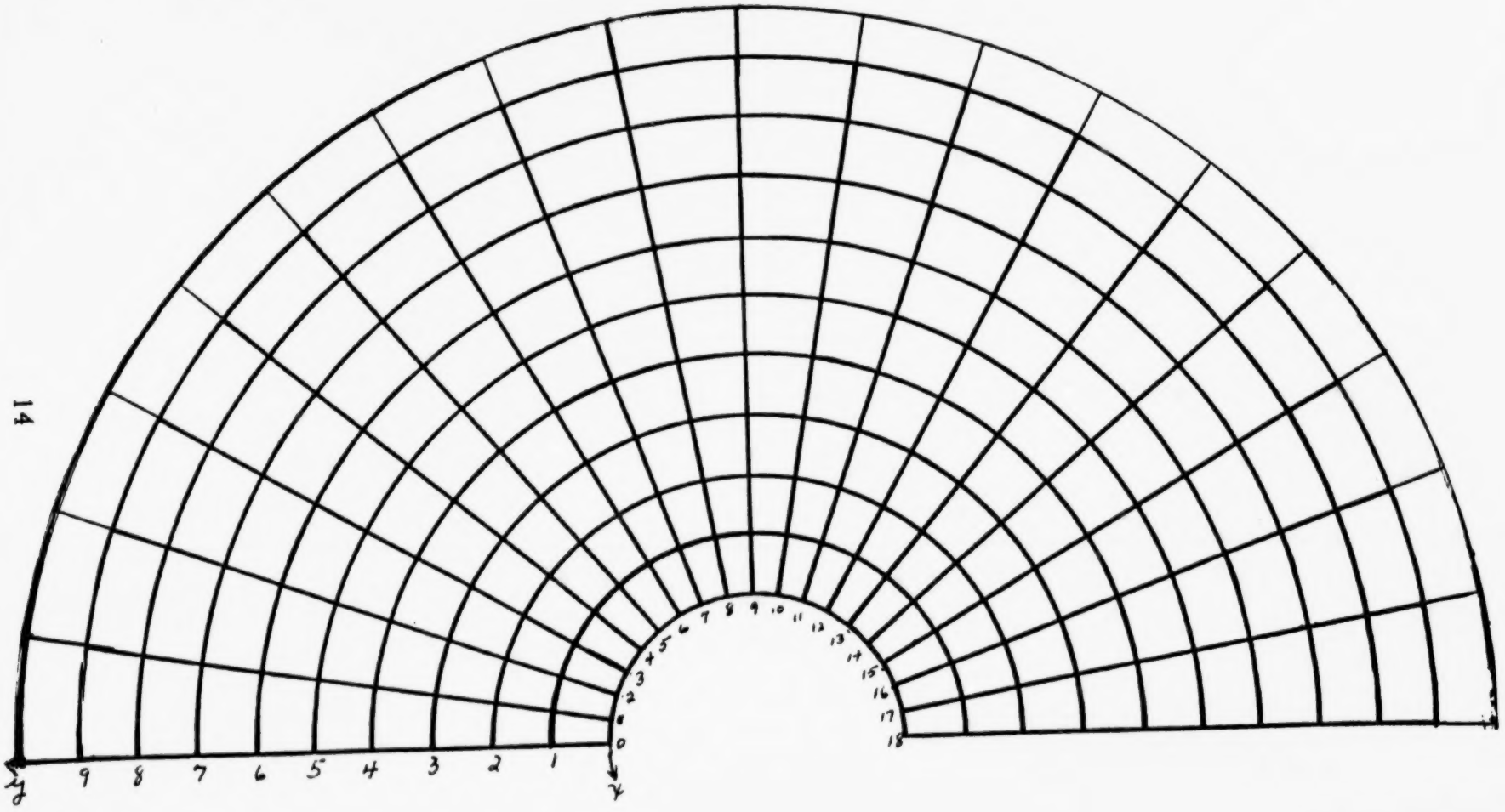
19

Activity C



Rectangular Grid

Cylindrical Graph Paper



14

ONE-DIMENSION: STRING ALONG

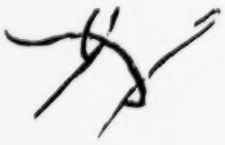
Objective: To use ratios and metric measurement to find relationships of parts of the body

Materials: a piece of string about a meter in length

Activity
Directions:

1. Find the distance around the thumb with the string.

2. Double the length of the distance around the thumb. It should be approximately the distance around the wrist.



3. Double the length of the string which is the distance around the wrist. This new length should be approximately the distance around the neck.

4. Double the length of the string again. This length is about the distance around the neck.

Extension
For each length, measure the string lengths on a metric ruler and compare the numerical measurements. Investigate the history of measurement before standard units: the cubit, the span, the hand, the foot, and etc.

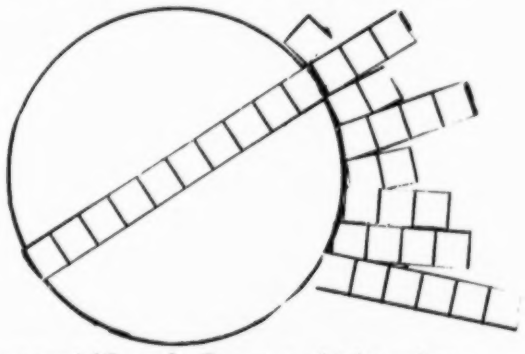
Going Around in Circles

Objective: To relate the diameter and circumference of a circle to π

Background

It was discovered very early in the history of geometry that for any circle, the ratio of the circumference to the diameter is always the same. The fixed ratio, or comparison, is denoted by the Greek letter "pi" (π).

** Each of the following activities can be used independently and necessary materials are noted within the activity.



Activity A

Materials:

Cuisenaire rods
compasses
calculators

Directions

Open a compass to the width of a yellow rod (5 cm). Draw a circle using this radius.

1. What is the diameter of this circle? _____
   Find the rod whose length is equal to the diameter of this circle and position it across the circle. Use any ten rods, and starting at the intersection of the diameter and the circle, lay these ten rods tightly together side by side so as to start a "sunburst" pattern.
2. What fractional part of the sunburst seems to be completed with ten centimeter rods? How many centimeter rods do you think it will take to surround the circle?
3. Complete the pattern. How many rods did you actually use for the circumference?
4. Use a calculator to find the ratio of the circumference to diameter (10) for this circle.

   Enter [circumference] [÷] [10] [=] Answer: _____
   The circumference is approximately three times the diameter. 3.1 is an approximation for this ratio to tenths. The consistent numerical ratio of circumference to diameter is known as "pi", usually with the symbol.

5. Repeat this experiment by drawing a circle with a radius equal to one orange rod (10 cm), which makes the diameter 20 cm (2 orange rods). After using the rods to complete the circumference, find the ratio C÷d.

Enter circumference ÷ 20 - Answer:_____

Does this number give you an approximation close to your first value for π ?

Activity B

Materials :

Paper plates, string with minimum stretch, centimeter ruler or measuring tape

Directions

1. Measure the circumference of the circular plate. You can do this by measuring a piece of string that surrounds the plate, or you can roll the plate one full rotation against a tape measure.
2. To find the approximate circumference of the circle, use the string to go around the circle. (Don't worry if your measurement of the circle is slightly different from another person's measurement.)
3. Find the diameter of the circle by folding the paper plate in half. The diameter is the line (fold) that cuts the circle into two equal parts.
   Measure the length of the diameter with the ruler:
4. Use a calculator to find the ratio of circumference to diameter for the circular plate.

   Enter Circumference ÷ diameter - Answer _____

The ratio C÷d is pi. The circumference of the circle is exactly π times the diameter of the circle. Depending on the accuracy we want, we say that the circumference is approximately 3 times the diameter, or 3.1 times the diameter, or 3.14 times the diameter, or...approximations could go on endlessly! The approximation for π is usually carried out to hundredths (2 decimal places).

With four other students, carry out the above directions. Complete the chart below for each person and follow the directions to average the C÷d results for all. Does this average for π gives an even closer approximation to 3.14?

	Circumference (Measurement of string around circle)	Diameter (measurement of fold line)	C÷d
1.	_____	_____	_____
2.	_____	_____	_____
3.	_____	_____	_____
4.	_____	_____	_____
5.	_____	_____	_____

Finding the average

1. Add the numbers in the C÷d column:
2. Divide by the number of entries listed in that column.
   The average for π is _____.

Activity C

Materials:

Several circular objects: different-size cans, records, etc.
string with minimum stretch; ruler or tape measure
newspaper, newsprint, or construction paper

Directions

Draw outlines of several different circular objects on newspaper or other paper. Cut out the circles, and label them with the object each matches.

Make a table to note the following information for each of the objects: Name of Object, Diameter, Circumference, C÷d.

Find the diameter by folding the circles in half. Measure the diameter. Measure the circumference for each circle. (You can do this by measuring a piece of string that surrounds the circle, or you can roll the circle one full rotation against a tape measure.)

Use a calculator to find the ratio C÷d. Enter [C] [÷] [d] [=]
Complete the chart for each object. Look at how each ratio C : d compares to 3.14, the approximation for π to hundredths.

Find the average of the ratios in the C÷d column. Use these steps:

1. Add the entries in the C÷d column.
2. Divide by the number of entries listed in that column.
   To hundredths (2 decimal places), the average is:
   Did this average give you a value for π close to 3.14?

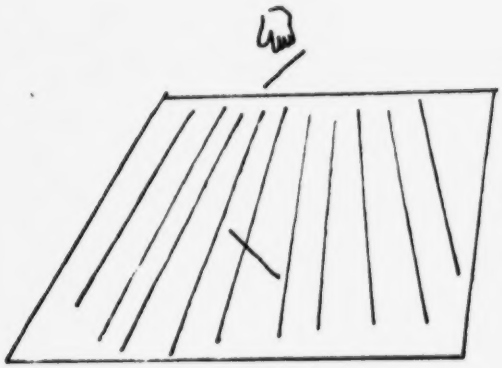
Activity D

Materials:

A floor with boards or tiles (Square-tile design remnants are available at linoleum stores)

A thin stick, such as a toothpick or "pick-up" stick, cut to the width of the boards or tiles

Directions



Drop a stick at least 50 times on a floor made of boards or tiles.

Make a chart to record the number of times the stick touches the lengthwise crack. Your chart should show the results of this experiment.

("Touched" -- "Did Not Touch") for trials 1,2, 3, ... 50.

1. Double the number of times the stick was dropped: _____
2. Count the number of times the stick touched a lengthwise crack. Answer _____
3. Divide the answer to #1 by the answer to #2_____

Although it takes advanced mathematics to explain why this occurs, your answer should be close to 3.14--or, pi!

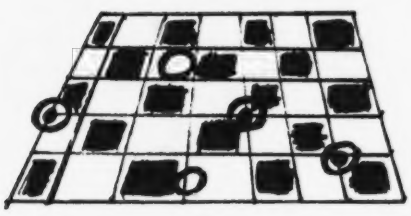
Activity E

Materials:

9" diameter paper plates on a floor with 9" square tiles; 6" diameter paper plates on a floor with 6" square tiles, etc.

Directions

Toss five paper plates onto a tile floor. Count the number of plates that cover a four-corner intersection. Record the count for Trial 1 in the table below. (The illustration shows three plates of five tossed covering a four-corner intersection.)



Record the count here:

Trials	1.	2.	3.	4.
No. Covering Corner	___	___	___	___

Repeat this procedure three more times for a total of 20 plates tossed.

Count the number of plates that covered a four-corner intersection each time. Record each count above.

The total number of plates that covered corners for all four trials is:

_____

Quadruple the number of plates that covered a corner:
Divide this number by the number of plates tossed (20).
Answer: _____

Is the value close to 3.14? Toss more plates to see if the answer comes closer to 3.14.

(Reference for Activity A: Urion, David K. "Using the Cuisenaire Rods to Discover Approximations for Pi." Arithmetic Teacher, 7, No. 24, December, 1979, p. 17)

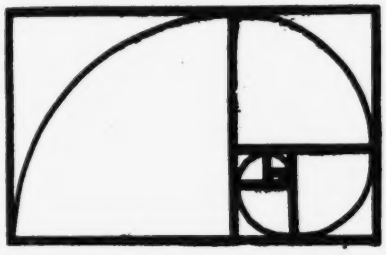
THE GOLDEN RATIO

Objective: To develop an understanding of the golden ratio

Background

The number sequence, 1,1,2,3,5,8,13,21,34,55,89 ... , constitutes the famous Fibonacci sequence in which each term is the sum of the two preceding terms. As the progression continues, the ratio between any term and its successor approaches the ratio of the golden section, 1 to 1.61. The ratio is noteworthy or "golden" because two dimensions in that ratio bear the unique relation to one another that the smaller is to the larger as the larger is to the sum of them both.
For example, 13 is to 21 as 21 is to 34.

The activities can be done independently and the necessary materials for each activity are listed accordingly.



Activity A

Materials:

graph paper, 10 units per inch
or worksheet

Directions

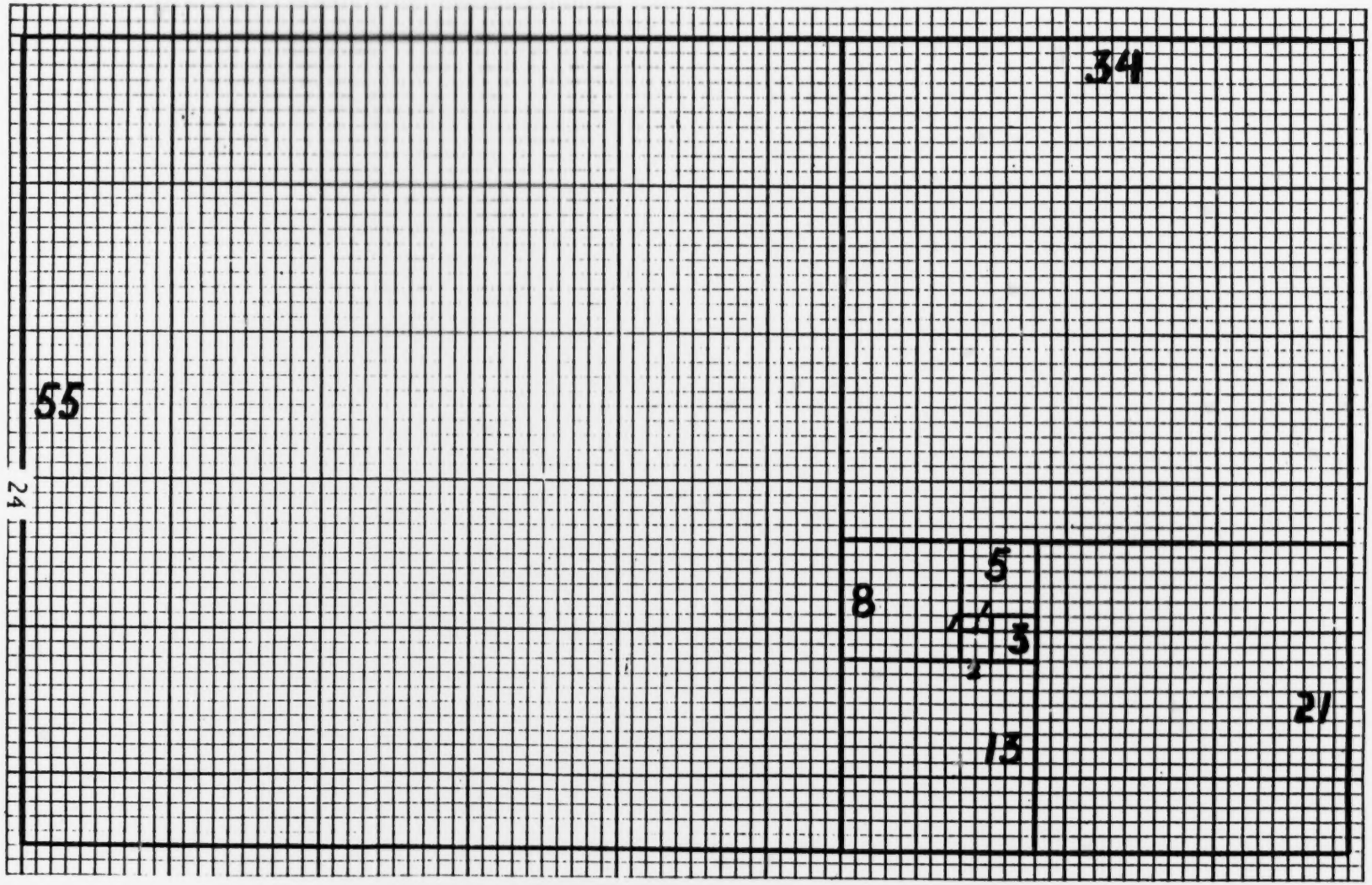
Hold the graph paper horizontally.

1. Draw a rectangle 55 units wide and 89 units long.
2. Divide the rectangle into a series of squares. Draw line segments into a series of squares whose sides are 55, 34, 21, 13, 8, 5, 3, 2, 1, and 1, respectively. (See included diagram.)
3. Starting with the largest square , carefully follow the diagram. Draw a quarter circle with a compass. A continuous spiral curve is formed.
   Based on the Fibonacci sequence , the resulting curve is similar to the chambered nautilus.

Extension A

1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 ... one number in the Fibonacci sequence is 89. Using the calculator, find the value of the reciprocal of 89 to six decimal places. What do you notice about the result?

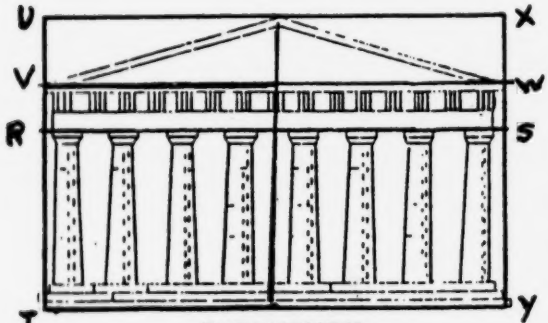
Can you find any other patterns using the reciprocals of the other numbers in the sequence?



Golden Rectangle.

Extension B

The ancient Greeks prized a certain kind of rectangle because of its shape. It occurs in much of their art. It seems pleasing in its aesthetic or artistic qualities. The front of the Parthenon fits almost precisely into the Golden Rectangle.



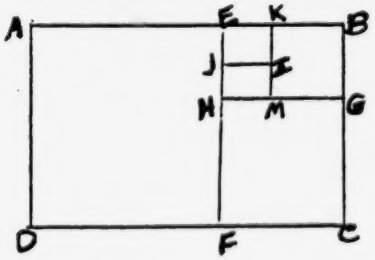
PARTHENON

The front of the Parthenon illustrates the presence of the Golden Ratio:

$$\frac{RT}{RU} = \frac{UT}{RT}$$

$$\frac{RU}{RT} = \frac{RV}{VU}$$

Slice a square ADFE off one end of the rectangle. A rectangle FCBE is left which is similar to the original. The process can go on forever.



Squares: AEFD, HGCF, KBGM, EKIJ

Rectangles: ABCD, EBCF, EBGH, HEKM

ABCD ~ EBCF, EBCF ~EBGH, EBGH~ HEKM

For discussion:

1. Explain in mathematical terms what a golden rectangle is.
2. What is the golden ratio? What is the connection between the golden rectangle and the golden ratio?
3. Can a very large and a very small rectangle both be golden rectangles?
4. Give three sets of dimensions which could be sides of a golden rectangle.
5. Give examples of rectangles that are around you that appear to be near-golden or golden.

Activity B

Background

In the book, Le Modular, which is an exposition of the Golden Ratio, LeCorbusier suggested in his work that the ratio of the height of a man to the distance from his navel to his feet is approximately the golden ratio.

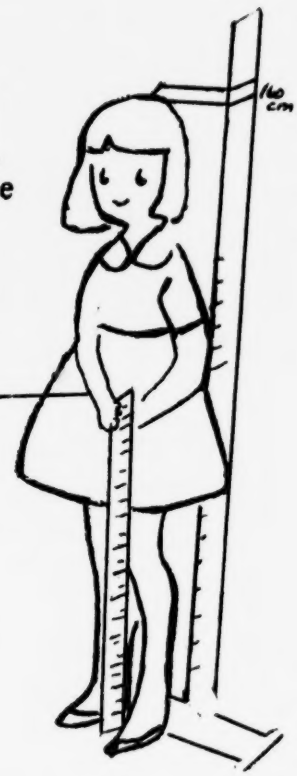
Materials:

metric measure stick (2 meters in length)
a metric ruler
calculators

Directions:

1. Each student is to measure his/her height (head to toe) and also, his/her height from the floor to his/her navel.
2. Use the calculator to divide the head to toe height by the floor to navel height.

$$\frac{160\,cm}{100\,cm} \approx 1.6 \qquad \frac{\text{head to toe}}{\text{floor to navel}}$$



3. Find the average of all of the ratios of the entire class.

Answers

Extension A

1. The reciprocal is 0.011235.
2. Answers will vary

Extension B

1. A rectangle with dimensions such that if a square is sliced off one end of the rectangle, a rectangle is left which is similar to the original.
2. The ratio of the sides of a golden rectangle.
3. yes
4. 10" x 16", 5 cm by 8 cm, 12m x 20m
5. picture frames, 5"x8" index card, proportions in a painting

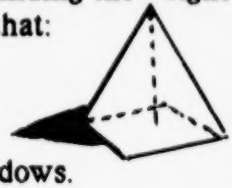
THE SHADOW FROM THE NILE

Objective: To use ratio and proportion to determine the height of tall objects through indirect measurement

Materials: Each group will need: meter stick, pencil and paper, metal measuring tape

Historical Background

Thales (Tha'lez) is called the father of Greek geometry. He lived in Miletus in Asia Minor about 600 B.C. The height of the pyramids of Egypt was unknown at the time. When Thales made a visit to Egypt he was able to surprise the king, Amasis, by finding the height by means of shadows. The method uses proportions such that:



The ratio of the heights of two vertical objects is equal to the ratio of the lengths of their shadows. Begin by looking at how Thales determined this conclusion.

Activity: The Shadow Method of Indirect Measurement

1. A. As the sun rises or sets in the sky, the length of the shadows cast by vertical objects changes. However, at any one time, the ratios found by comparing the heights of these objects to the length of their shadows are all equal. The equal ratios form a proportion.

Suppose a meter stick (100 cm) casts a 50 cm shadow at 10 a.m.

$$\frac{\text{Height of stick}}{\text{Length of shadow}} \qquad \frac{100\text{cm}}{50\text{cm}} \qquad \text{or, 2 to 1.}$$

At 10 a.m. the ratio of the height of any perpendicular object to the length of its shadow will be 2:1. You do not have to climb a nearby tree to find its height. What would be the height of a nearby tree that casts a shadow of 250 cm? _____

1 B. At a different time of day, the ratio of meter stick to shadow is 100:40. The shadow of the tree was 200 cm.

i. Compare the meter stick to its shadow.
How many times bigger is the meter stick? _____

ii. Predict the height of the tree compared to its shadow.
Has the height of the tree changed? _____

2. Look at the comparison between the shadows of the meter stick and the tree at both times of the day.

$\frac{100cm}{50cm} \rightarrow \frac{?}{250cm}$ $\frac{100cm}{40cm} \rightarrow \frac{?}{200cm}$

A. How many times bigger is the tree's shadow in each case? _____

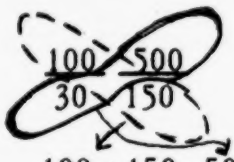
B. The tree is just that many times as tall as the meter stick. The height of the tree is _____ times the height of the meter stick, or _____ cm.

3. Note to the teacher: Students should be familiar with solving equations before using the following method:

A. Another property of proportions is that the cross products are equal.

100 x 250 500 x 50

25000 = 25000

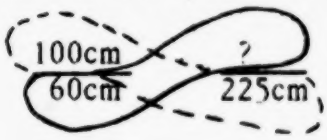


100 x 150 500 x 30

15000 = 15000

B. If the comparisons between heights of objects or lengths of their shadows are difficult numbers to compute mentally, cross products can be more useful in finding the missing measurement.

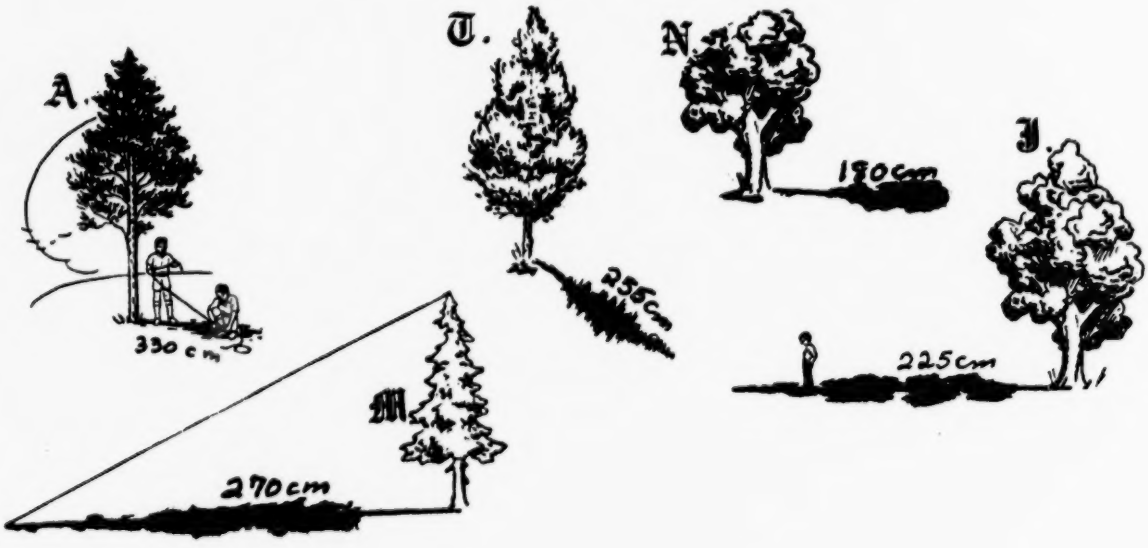
Example: Find the height of a tree that casts a shadow of 225 cm if a nearby meter stick casts a shadow of 60 cm.



Then, 60 x ? = 22500
(and dividing 22500 by 60)
? = 375 cm

Exercises

1. A meter stick casts a shadow of 30 cm. Find the heights of nearby trees A, M, T, N and J based on the lengths of their shadows. Use any application of the shadow method described above.



2. Find your height by the shadow method described above.

3. Use the shadow method to find the height of your school's flagpole.

Answers

1A. 500 cm

Bi. 2 1/2

Bii. 500 cm
The height of the tree has not changed.

2A. In each case the height is five times bigger.

B. 5 times bigger...500 cm.

Exercises

1. Heights of trees:

A = 1100 cm M = 900 cm
T = 650 cm N = 600 cm
J = 750 cm

2. Answers will vary.

3. Answers will vary.

DOES IT MEASURE UP?

Objective: To examine the relationship among lengths of sides forming a triangle
To examine the relationship among lengths of sides forming a right triangle

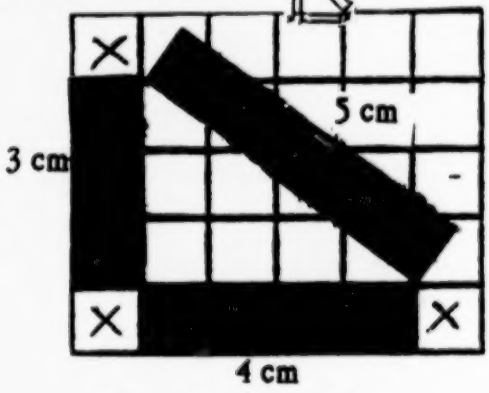
Materials: One box of Cuisenaire Rods per student
a sheet of centimeter-square graph paper

Activity

Against the bottom edge of the graph paper, arrange the ten basic Cuisenaire Rods in ascending order of length, like a staircase. Beneath each rod, note its length in cm. Continue the staircase through the number 13, adding on to "10" rods to do so.

Using rods matching the lengths below, see if the rods can form a triangle. Rods should remain fixed along a corner edge, like so --not overlapping.

When the rods do form a triangle, see if you can position the rods on the graph paper so that they form a right triangle, as in the illustration.



1. Complete the chart below.

Lengths in cm			TRIANGLE?	RIGHT TRIANGLE?
a	b	c	Yes or No.	Yes or No.
2	3	6	_____	_____
2	4	6	_____	_____
2	5	6	_____	_____
3	4	6	_____	_____
3	4	5	_____	_____
4	5	10	_____	_____
5	6	10	_____	_____
6	8	10	_____	_____
5	8	13	_____	_____
5	10	13	_____	_____
5	12	13	_____	_____

2. Can you predict when three rods will make a triangle? _____
3. Name another three lengths that would form a triangle. _____
4. The notation "a^2" means a x a, or a number times itself.

Use a calculator to show that the relationship $a^2+b^2=c^2$ holds for the three combinations that you found formed right triangles:

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
—	—	—	—	—	—	—
—	—	—	—	—	—	—
—	—	—	—	—	—	—

5. Select one of the combinations that you found did <u>not</u> make a right triangle and show that the relationship $a^2 + b^2 = c^2$ does not hold.

a	b	c	a^2	b^2	$a^2 + b^2$	c^2
—	—	—	—	—	—	—

6. Suppose two sides of a triangle measured 7 cm and 24 cm. Use the calculator to find the length of the third (longest) side that would form a right triangle.

Extensions

Investigate the contributions made by rope stretchers in ancient Egypt.

Do the following for those lengths that formed a right triangle: Cut graph paper into squares whose edges are equal in length to measurements a, b, c. Count the squares in "a^2" and "b^2" to see if they match the number of squares in "c^2". Cut the two smaller squares apart so that the pieces form the largest square.

Research the discoveries made by Pythagoras and his followers. Why did they wish to keep these discoveries secret?

<u>Reference</u>

Fuys, David and Tischler, Rosamond. <u>Teaching Mathematics in the Elementary School.</u>

ANSWERS TO QUESTIONS ABOUT "TRIANGLE INEQUALITIES"

1. Complete the chart below.

Lengths in cm			TRIANGLE?	RIGHT TRIANGLE?
a	b	c	Yes or No.	Yes or No.
2	3	6	No	No
2	4	6	No	No
2	5	6	Yes	No
3	4	6	Yes	No
3	4	5	Yes	Yes
4	5	10	No	No
5	6	10	Yes	No
6	8	10	Yes	Yes
5	8	13	No	No
5	10	13	Yes	No
5	12	13	Yes	Yes

2. Three rods make a triangle if the combined lengths of the two smaller rods is greater than the length of the largest rod.
3. Any a, b, c such that $a + b > c$.
4.

a	b	c	a^2	+	b^2	=	c^2
3	4	5	9		16		25
6	8	10	36		64		100
5	12	13	25		144		169

5. Anyone of the eight "no" answers in the last column of exercise one.
6. The largest rod would be 25 cm long.

HANDING IT OVER FROM 1-D TO 2-D

Objective: To approximate the perimeter and area of a figure

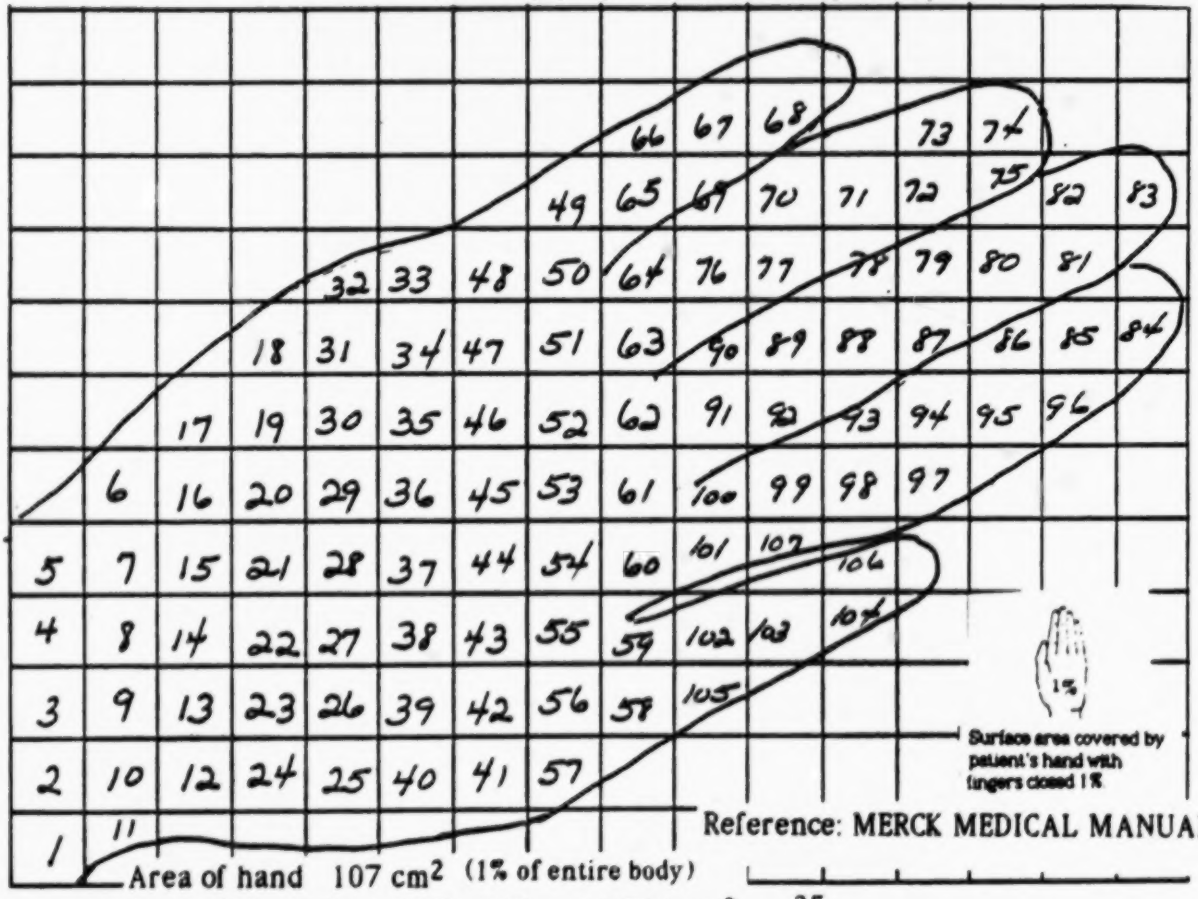
Materials:

string
centimeter rulers
centimeter square paper

Activity

Directions:

1. Draw an outline of the hand on centimeter square paper. Find the perimeter of the hand using string and the centimeter paper.
2. Area is a region or amount of surface. A region is a simple closed curve and its interior.
   Find the area of your palm by counting the squares. Count half a square or more as one square.
3. Did you know that the area of a person's hand is one percent of the surface area of the entire body?
   Find the total area by multiplying the area of the palm by 100.



Reference: MERCK MEDICAL MANUAL

Area of hand 107 cm^2 (1% of entire body)

Area of the body is 107 x 100 or 10 700 cm^2

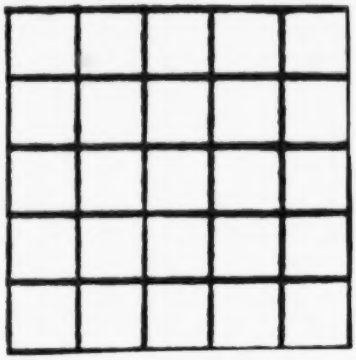
35

THE PROBLEMS WITH AREA

Objective: To use problem solving skills to solve two-dimensional geometry problems

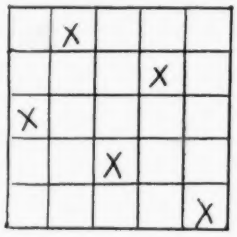
Materials: One copy per child of these two activity sheets
five cubes, 1 cm on an edge, or other appropriate markers

1. Place five cubes on the squares so that no two cubes are on the same horizontal, vertical, or diagonal line. After getting a solution to the puzzle, color the square under each cube.

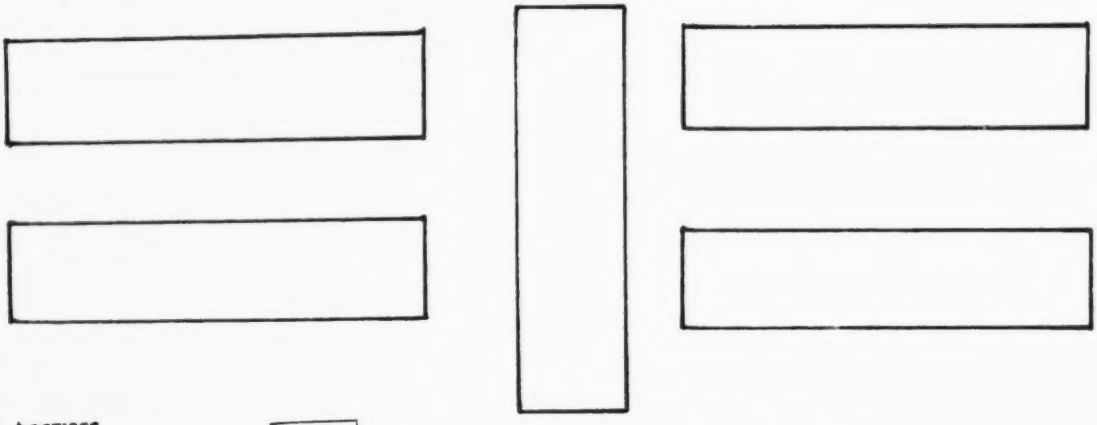


Answer

1.

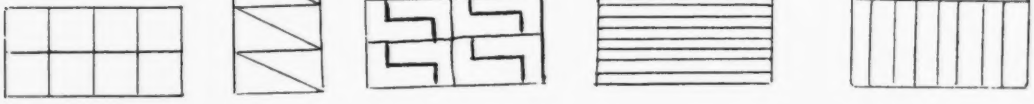


2. Rob has 5 pure chocolate candy bars that he wants to share with friends. Show five different ways to cut a rectangular candy bar into eight pieces, making sure that each piece of a bar is the same size and shape.



Answers

2.



3. Figure 1 shows a board 15 cm by 2 cm. Figure 2 shows a hole 10 cm by 3 cm. How can you cut the board into two pieces so that it covers the hole completely?

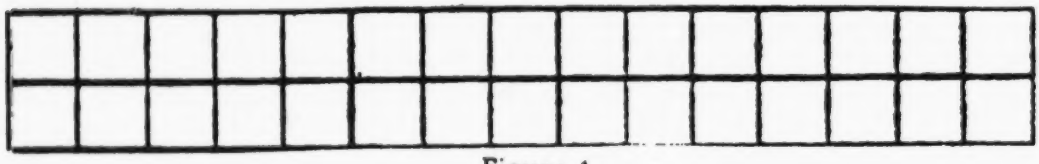
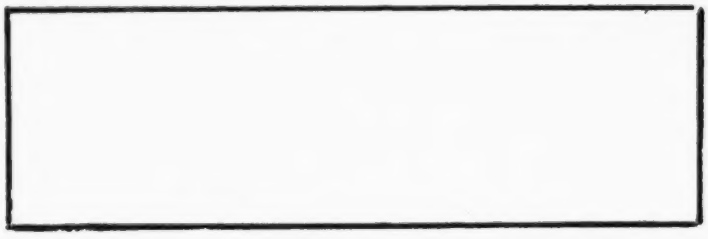
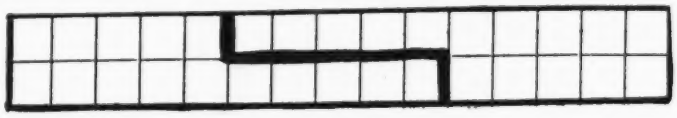


Figure 1

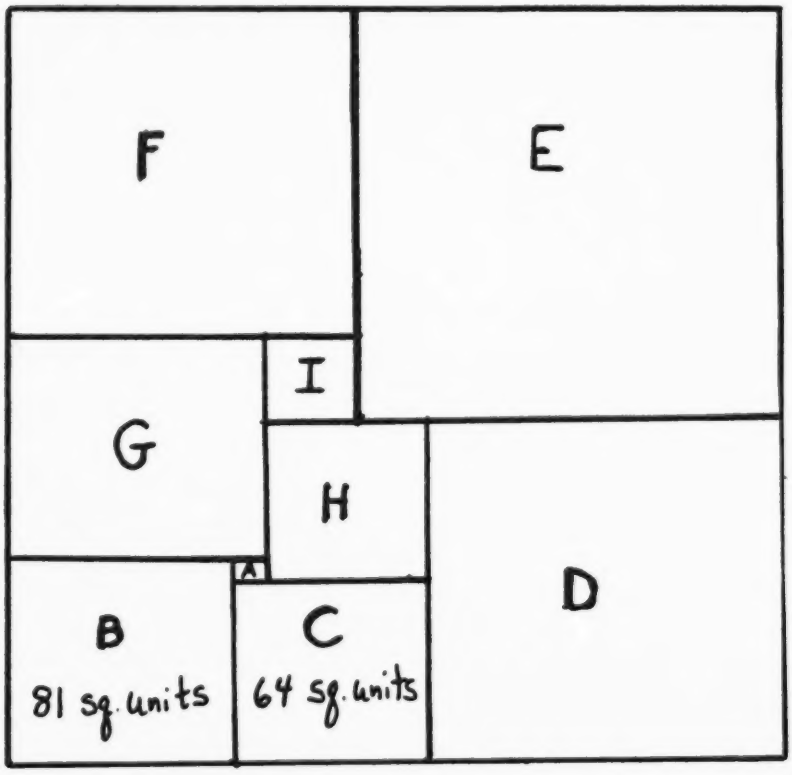
Figure 2



Answer 3.



4. From the information given, find the area of each of the square regions in the figure below:



Answers

4.

Letter	Measure of Side	Area
A	1	1
B	9	81
C	8	64
D	15	225
E	18	324
F	14	196
G	10	100
H	7	49
I	4	16

FROM 2-D TO 3-D: ALL BOXED UP

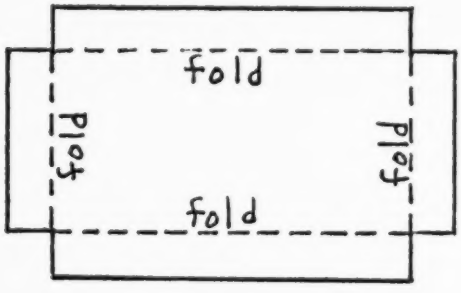
Objectives: To apply the concept of volume to rectangular prisms
To justify the formula for the volume of a rectangular prism
To organize data in tables and graphs
To interpret data from tables and graphs

Materials: centimeter square paper
ruler, pencil, tape
scissors
one (1) cubic centimeter blocks

Activity A: Volume of a rectangular prism

Using the centimeter square paper, cut out two congruent rectangles. This activity is illustrated using 15 cm by 12 cm rectangles.

Using one of the rectangles, form an open box by cutting a 1 cm by 1 cm square from each corner of the rectangle and folding the sides up. This three dimensional shape is a rectangular prism. Tape securely at each corner.



Fill the box with 1 cubic centimeter blocks. Count the blocks. This is the volume of the box in cubic centimeters.

Using the second rectangle, form another box by cutting a 2 cm by 2 cm square from each corner of the rectangle. Find the volume of this rectangular prism by filling it with 1 cubic centimeter cubes.

Using these two boxes, generalize the formula for the volume of a rectangular prism.

Activity B: Maximum volume of a rectangular prism

Continue to use a rectangle congruent to the one in Activity A.

Complete this table for each size corner square. If necessary, make a box and fill it with blocks for each entry.

Square Size	Length of the box	Width of the box	Height of the box	Volume of the box
0 by 0	_____	_____	_____	_____
1 by 1	_____	_____	_____	_____
2 by 2	_____	_____	_____	_____
3 by 3	_____	_____	_____	_____
4 by 4	_____	_____	_____	_____
5 by 5	_____	_____	_____	_____
6 by 6	_____	_____	_____	_____

Generalize a formula for the volume of these boxes.

Note to the teacher: If n represents the length of the side of the corner squares then for the 15 cm by 12 cm rectangle

Length (l) - 15 - (2 x n)
Width (w) - 12 - (2 x n)
Height (h) - n
Volume (V) - l x w x h - (15 - 2 x n)(12 - 2 x n)(n)

By referring to the table, predict the square size that will produce the box with the largest (maximum) volume.

Extensions

On 1/4" graph paper, graph the values for the sides of the corner square on the horizontal axis and the volume of the rectangular prism on the vertical axis. Use the graph to approximate the square size that will yield the largest (maximum) volume.

Calculate the volume for this approximation. Refine your approximation based upon this calculation.

Construct a table of values for the volume for corner square size from 2 cm to 3 cm in steps of .1 cm.

Students who are capable of writing a computer program may be able to produce a table of successively smaller incremental values in order to obtain better approximations.

Investigate procedures for finding the exact maximum value for the volume of a variety of three-dimensional figures.

ANSWERS

Activity A:

The formula for the volume (V) of a rectangular prism is the product of the length (l), the width (w) and the height (h).

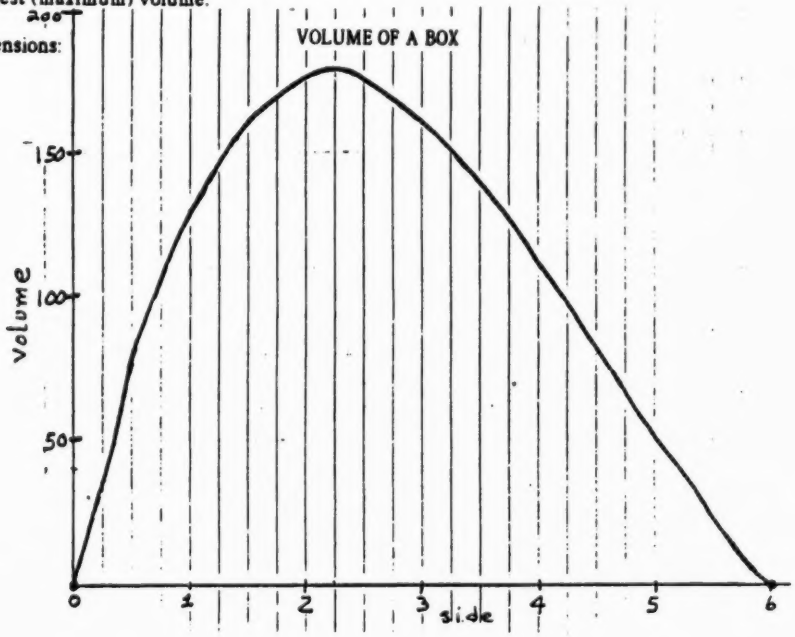
V = l x w x h

Activity B:

Square Size	Length of the box	Width of the box	Height of the box	Volume of the box
0 by 0	15	12	0	0
1 by 1	13	10	1	13
2 by 2	11	8	2	176
3 by 3	9	6	3	162
4 by 4	7	4	4	112
5 by 5	5	2	5	50
6 by 6	3	0	6	0

Conjecture: The 2 cm by 2 cm square corner produces the box with the largest (maximum) volume.

Extensions:



From this graph, the approximate square size for the largest volume is about 2.25 cm with a volume of 177.2 cubic centimeters.

Square Size From 2 cm to 3 cm in Steps of .1 cm

Side of the square	Volume of the box
2. by 2.	176.
2.1 by 2.1	176.9
2.2 by 2.2	177.2
2.3 by 2.3	177.0
2.4 by 2.4	176.3
2.5 by 2.5	175.
2.6 by 2.6	173.3
2.7 by 2.7	171.1
2.8 by 2.8	168.4
2.9 by 2.9	165.4
3. by 3.	162.

SHAPING UP WITH COLOR

Objective: To analyze patterns for plane and solid figures through the use of color

Materials: Worksheets A and B for each child
staplers
scissors
3 strips of different color paper per child, each strip being approximately 6 cm wide and 20 cm long
Fun Tack, or other plastic adhesive

Background

In tiling a plane, color is often used to separate the pattern pieces into distinct shapes. Let's agree that two tiles that share a common edge may not be colored the same way. The "tiles" may be regular geometric pieces, or irregular shapes that interlock, as in a map. M. C. Escher used geometric shapes as the basis of his intriguing tile designs, and their coloring helped us appreciate the complexity--and simplicity--of the designs. Similarly, the different planes ("faces") of a solid can be distinguished through different colors. Mathematicians have found that four colors are sufficient to tile any surface, plane or solid, in order for the distinct shapes to be visible.

Have students layer 3 strips of different color paper beneath the design centered on Worksheet A. Staple in at least 6 places. Cut out the individual shapes forming the design. Including the white shapes, there will be 16 parallelograms, 24 squares, 16 triangles, and 12 hexagons.

Activity A

Find the least number of different color tiles needed to fill each of the figures on the Worksheet B. Remember that tiles that share a common edge may not be colored the same way.

What is the least number of colors needed to tile:

the pattern of squares? _____ the pattern of parallelograms? _____

the pattern of triangles? _____ the pattern of hexagons? _____

Activity B

Cut around the heavy lines of each of the figures.

1. Which two of the patterns seem similar? _____
2. Experiment until you find the two patterns which can be folded into solids. Name the solids formed. _____
3. Explain why the two remaining patterns cannot be folded into solids.

Activity C

Work with the two patterns which can be folded into solids. Note how the edges come together in forming the solid. Use the rule that two faces that share a common edge may not be colored the same way. Use Fun Tack or another adhesive to experiment with the colored tiles so that no two tiles sharing a common edge are the same color.

What is the least number of colors needed to tile <u>each</u> of the solid figures?_____

Worksheet A

Extensions

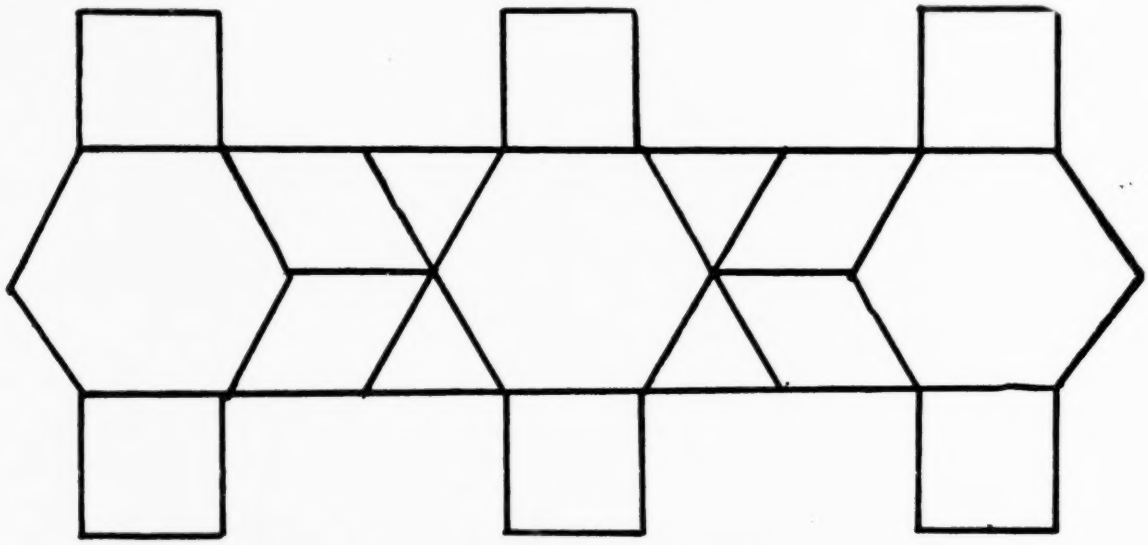
Use square tile pieces to discover other layouts that can be folded into cubes.

Read more about "The Four Color Map Problem" in the Mathematics volume of the Life Science Library (Bergamini, D. et al. New York: Time, Inc. 1963)

The tile pieces can be used for fraction relations by covering the hexagon with parallelograms and triangles. For more on this, consult Cuisenaire activities on Pattern Blocks and fractions.

Tangrams can be used for fraction relationships, conservation of area, and picture designs.

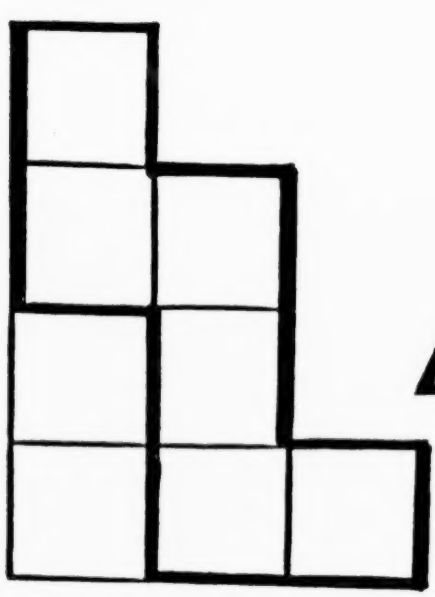
The design on this page may be considered a semi-regular tessellation, meaning that a combination of figures interlock to tile the plane. Extend the border pattern vertically as well as horizontally; you may add new shapes to do so.

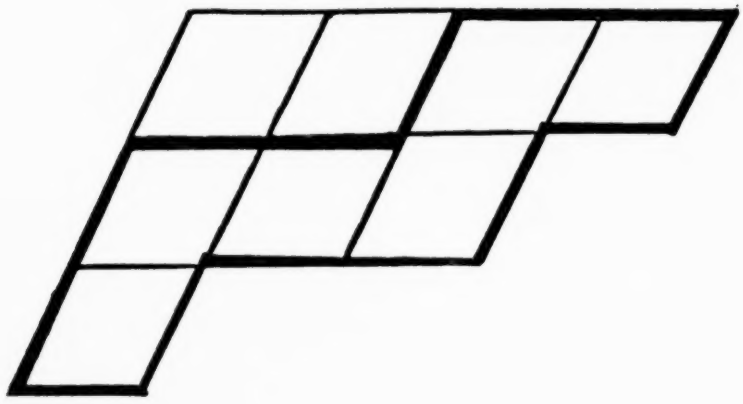


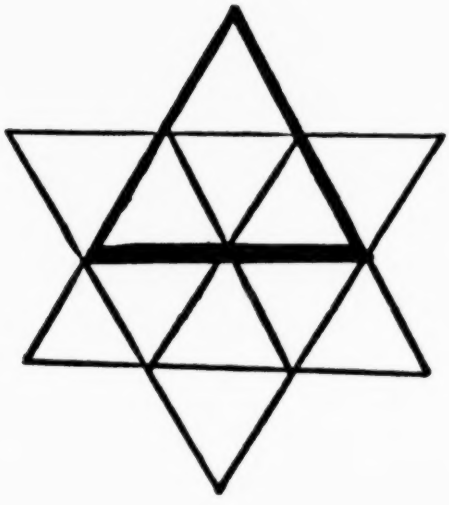
The design as is may be examined for symmetry. Coloring the design will help to emphasize the vertical and horizontal symmetry. Designs having symmetry are examples of transformations known as reflections.

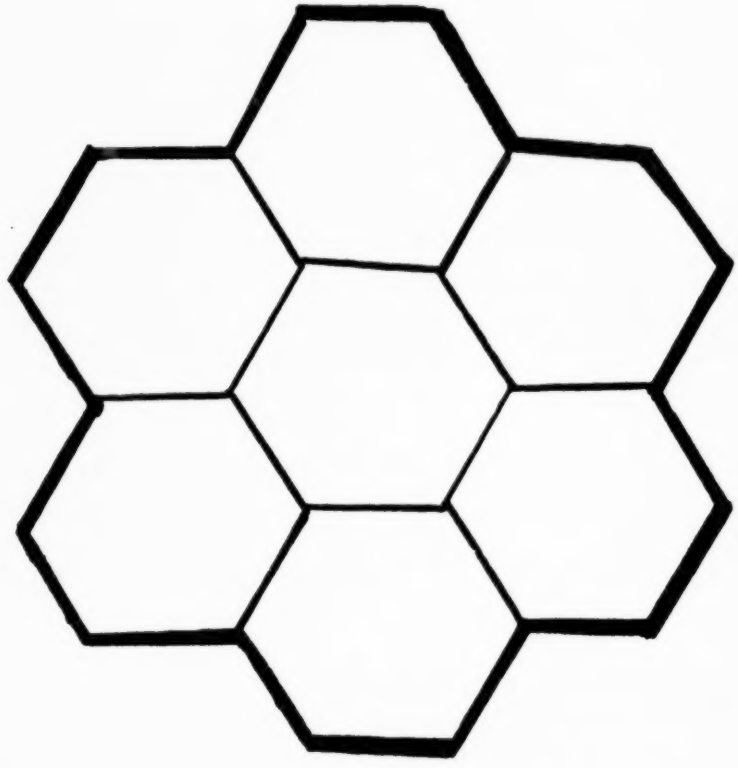
Find the two pairs of shapes which, if interchanged in the pattern, it would transform it from a reflection to translation (slide), --that is, a regularly repeating pattern. Find the seven connected figures which could be used for a slide-repeating design. Trace them on to a clean sheet of paper, and with a ruler, complete at least two more multiples of the pattern.

Worksheet B









Answers

A squares 2 parallelograms 2 triangles 2 hexagons 3

B 1 Squares & parallelograms <u>seem</u> similar
2 The squares form a cube, and the triangles form a tetrahedron (triangular pyramid).
3 The patterns do not form solids because there are gaps at the corners

C You need 3 colors for the cubes and 4 colors for the tetrahedron

PAPER PLATE GEOMETRY

Objective: To construct various polygons and a polyhedron

Materials: Circles nine inches in diameter cut from newspaper or construction paper. (Note: You can cut through multiple thicknesses of paper more easily if several layers are stapled together. Construction paper will allow more rigid solids.)

** Directions which follow are for the teacher to demonstrate to the class for their construction of the figures. The directions are accompanied by questions to lead the students to do critical thinking as well as visualization.

Preparation

Use one circle as a master. Locate its center by folding twice in half, so that two diameters intersect. With a protractor, mark off arcs of 120^0, corresponding to 12 o'clock, 4 o'clock, and 8 o'clock on a clock face.

Place a stack of circles beneath the master and use a compass point to pierce through the center, making sure each circle in the stack has a small hole. With a marker, make sure the edges of all the circles have a small dot at 12 o'clock, 4 o'clock and 8 o'clock, as in Figure 1.

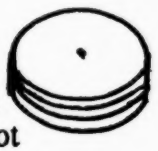


Figure 1

Activity

1. Distribute the circles to students. They have been given a plane figure which is not a polygon, but which will be transformed into several polygons, (straight-sided flat shapes) and even a solid polyhedron shape.
2. Enlarge the center hole with a pencil or pen point.
3. Turn the circle so that a dot appears at 12 o'clock. Indicate other dots at 4 o'clock and 8 o'clock. Fold the circle from 12 o'clock to 4 o'clock--if this is done properly the rim of the circle should just reach the center, as in Figure 2. What fractional part of the rim of the circle has been folded in toward the center? (1/3). Can you guess which polygon will be formed? (triangle)

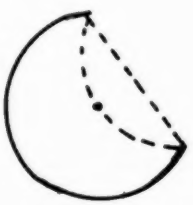


Figure 2

4. The next fold creases another third of the circle towards the center, from the markings for 4 o'clock to 8 o'clock, or 8 o'clock to 12 o'clock (See Figure 3). Emphasize that this new fold and the previous fold must meet at a sharp corner (called a vertex). Children will often remark as to the wedge-shaped or conical appearance of the result that reminds them of pizza pie slices or ice cream cones. What fractional part of the circle's rim is now folded in to the center? (2/3).

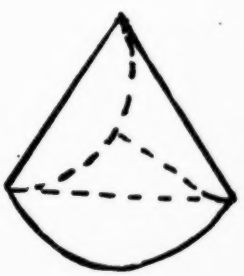


Figure 3

5. Fold the remaining 1/3 of the circle's rim toward the center, obtaining an <u>equilateral triangle</u>, as shown in Figure 4.

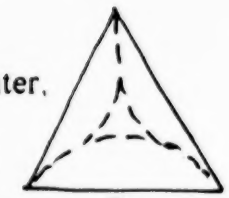


Figure 4

6. Do you see a way to convert the triangle into a six-sided hexagon?
   To obtain a hexagon, fold each vertex of the triangle to the center, as in figure 5. The figure obtained is a <u>regular hexagon</u>, because all of its six sides and all of its six angles are <u>congruent</u>.

Figure 5

7. Do you see a way to change the hexagon into a trapezoid?
   To make a trapezoid, fold along any one of the three central diagonals shown in Figure 5. The result is the trapezoid shown in Figure 6.

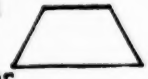


Figure 6

8. In a <u>parallelogram</u>, opposite sides are congruent. Fold the lower right vertex on the long side to the upper left vertex on the short side, as in Figure 7. You have obtained the parallelogram defined as a <u>rhombus</u>, because all four sides of the parallelogram are congruent.

Figure 7

   Unfold the rhombus to the trapezoid, and refold it from the <u>lower left</u> vertex on the long edge to the upper right vertex on the short edge, as in Figure 8. You will need this extra crease to make the solid which follows.

Figure 8

9. The last figure we will make is a three-dimensional space figure--a regular tetrahedron, which is a pyramid of four congruent faces.

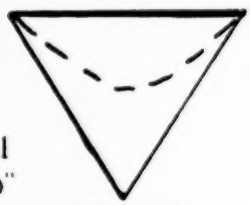


Figure 9

   A. It is important to unfold the polygon to the large original triangle with the top-most flap of the triangle at the "top" corner, as in Figure 9.

Figure 10

   B. Fold the bottom vertex to the center, creating a trapezoid; then fold the short bottom edge of the trapezoid to the long top edge. (See Figures 10 and 11).

Figure 11

   C. The students should be able to see 5 triangles, which you may wish to label as a, b, c, d, and e, as in Figure 12. Since a tetrahedron is composed of four triangular faces, one will "disappear." The center triangle, c, will be the base of the pyramid; its two adjacent triangles, b and d, will be two of the lateral faces; and either of the two remaining triangles (a or e) should "pocket" into the other as we raise the pyramid shown in Figure 13. (This works easily if step 9A has been followed.)

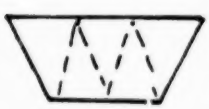


Figure 12

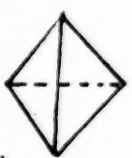


Figure 13

   D. Secure edges with masking tape.

Extensions

The resulting tetrahedron can be related to dice found in "Dungeons and Dragons". Another activity is to write the number 6, 7, 8, and 9 on the faces of the die and discuss which face we will consider as the outcome on the roll of a die. The outcomes on pairs of tosses can be multiplied, with the students tallying whether more odd or even products occur.

Since even products result from at least one factor being even, 3/4 of the 16 possible products will be even. The results can be confirmed by looking at the possible products in an array diagram where the possible factors are listed along each of the axes, and the resulting products placed in the boxes. (See Figure 14.)

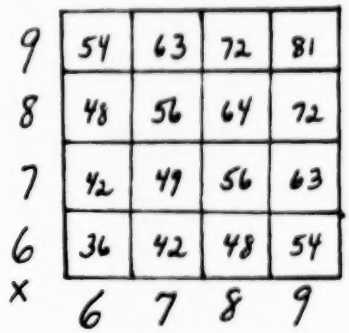


Figure 14

Children may enjoy the study of origami (Japanese paper-folding) at this point.

In making the hexagon, it is easy to see that each individual triangle makes up 1/6 of the hexagon, and that the trapezoid makes up 1/2 of the hexagon. What is the fractional relationship between the rhombus and the hexagon? For more on fraction relationships explored through tiling, consult Cuisenaire activities on Pattern Blocks and Fractions.

OF TOOTHPICKS AND MARSHMALLOWS

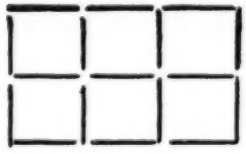
Objective: To reinforce visualization and organizational concepts for geometry in one, two and three dimensions.

Materials: Miniature marshmallows, or gumdrops, to represent points
Toothpicks, matchsticks, or pretzel sticks to represent line segments
Game mat for Activities 7 - 10
Recording sheets (grid paper made up of centimeter squares and equilateral triangles)

1. Two line segments are joined by a point. If they must form either 90 or 180 degree angles, there are only two possible arrangements:

In a plane, how many formations are possible with <u>three</u> line segments joined at two points at angles of 90^0 or 180^0? Draw the formations on centimeter square paper.

2. Make this diagram using 17 toothpicks. Record the solutions on centimeter square paper.

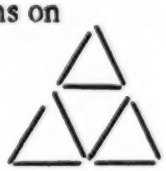


a. Find the 8 squares in the diagram.
b. Remove 4 toothpicks so that 3 squares remain.
c. Reassemble the diagram. Remove 6 toothpicks so that 2 squares remain.
d. Reassemble the diagram. Remove 5 toothpicks so that 3 squares remain. (Hint: The squares are not adjacent to each other, meaning they do <u>not</u> share a common side.)

3. Make this figure with 12 toothpicks. Record the solutions on equilateral triangle grid paper.

a. Remove 4 toothpicks so that 3 triangles remain.
b. Remove 4 toothpicks and use them to make three triangles.

4. Make this figure with 9 toothpicks. Record the solutions on equilateral triangle grid paper.

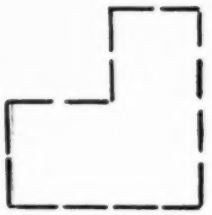


a. Remove 2 toothpicks so that 3 triangles remain.
b. Reassemble the diagram. Remove 3 toothpicks so that 1 triangle remains.
c. Reassemble the diagram. Remove 6 toothpicks so that 1 triangle remains.
d. Reassemble the diagram. Remove 4 toothpicks so that 2 triangles remain.
e. Reassemble the diagram. Remove 2 toothpicks so that 2 triangles remain.

5. Use 11 toothpicks to construct this figure. Record the original diagram and the solution on equilateral triangle grid paper.

Suppose you were looking at a picture of a house where the outside seems horizontally bricked and the inside seems vertically paneled. By moving one toothpick, can you make the house appear bricked inside and paneled outside?

6. Two farmers have land in this shape. Use 16 toothpicks to fence in the land.



   a. The first farmer wants to divide his land equally among his three daughters. Use 4 more toothpicks to divide the land into three congruent polygons (meaning figures having the same size and shape.) Record the solution.

   b. Return to the original diagram. The second farmer wants to divide her land equally among her four sons. Use 8 more toothpicks to form four congruent polygons. Record the solution.

For questions 7-10, use the playing mat made up of 16 square units, with A nd B marked in corners diagonal to each other. When trying straight-line paths, overlap sticks as necessary. You may only follow horizontal—and | vertical line segments; you cannot cut diagonally (\ or /) across corners.

7. On the accompanying diagram, place 4 markers on four squares so that no two markers are in the same line vertically, horizontally, or diagonally. Record your solution on cm square graph paper.

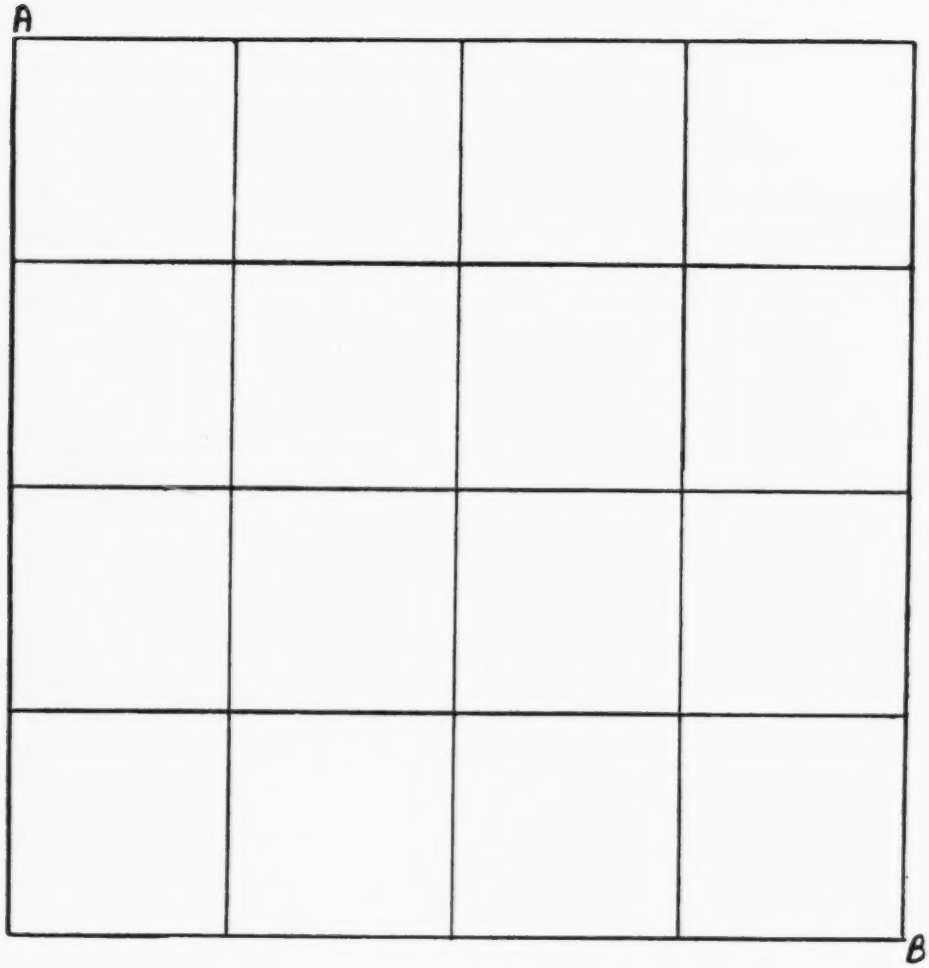
8. Place markers at A and B. If A and B meet at the center point of the diagram, then each would cover the same distance on straight line paths to get there. Can you find another point that would require A and B to travel the same amount of straight-line distance in order to meet? How many such points are there in all? Record your solutions.

9. Use 4 or more toothpicks to find as many ways as possible to divide the large square into two congruent polygons. (Record your solutions on cm square paper.)

10. a. Find the total number of 4-unit paths that A can take to get to the center. Record your solutions.

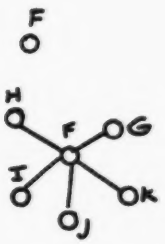
    b. Find the total number of 8-unit paths that A can take to get to the center. Record your solutions

PLAYING MAT



11. Research Buckminister Fuller and geodesic domes before following the instructions to complete a geodesic dome. To make this dome, you will need 11 marshmallows (or gumdrops) and 25 toothpicks.

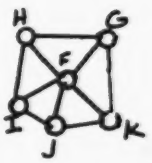
Instructions



a. Start with Point F. (A marshmallow represents a point).

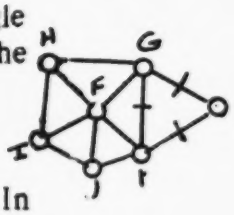
b. Connect five line segments (toothpicks) to point F to create a star pattern.

c. Place a marshmallow at the end of each line segment.

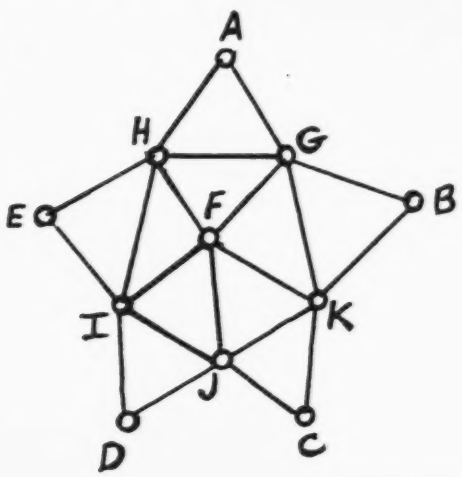


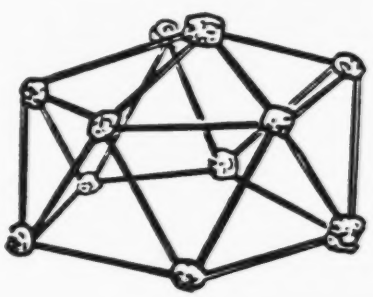
d. Create a pentagon by placing toothpicks between the outer-edge marshmallows.

e. Along each edge of the pentagon, form an equilateral triangle using one more marshmallow and two more toothpicks. (The illustration shows one of the five equilateral triangles.)



f. Attach 5 toothpicks between the outer points of the figure (In the diagram, attach toothpicks between A-B, B-C, C-D, D-E, E-A, so that point F becomes the peak of a geodesic dome.

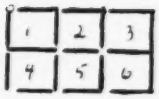




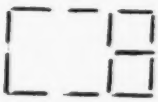
Answers

1

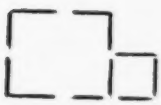
2. a.



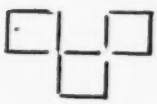
b.



c.



d.



3 a.

b.

4. a.

b.

c.

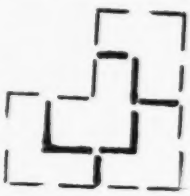
d.

e.

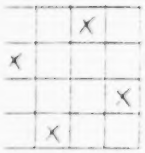
5

6. a.

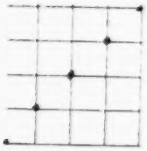
b.



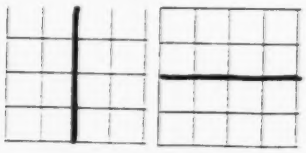
7



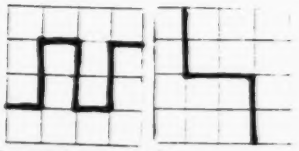
8



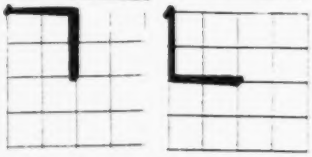
9



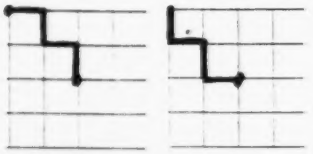
10.



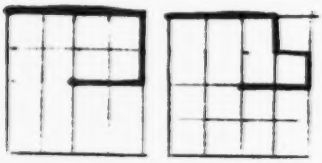
A

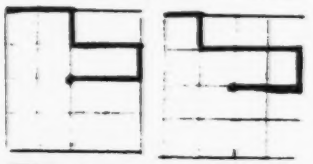


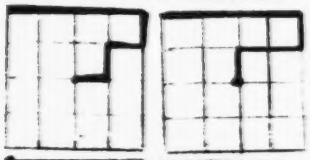
Examples



10B







A METER CUBED

Objectives: To develop an understanding of the concept of a metric unit of measure
To estimate and measure lengths using centimeters and meters
To find the volume or the capacity of a cubic meter
To represent weights using kilogram units
To construct a cubic meter

Materials:

a metric scale
12 meter sticks with connecting material to join the corners
a worksheet

The volume of a geometric figure refers to the amount of space the figure occupies or encloses.

Activity

1. Use the 12 meter sticks to assemble the cube. The length, the width, and the height are equal to one meter.
2. Determine how many cubic centimeters are in one cubic meter.
3. Each student is to weigh himself/herself on the metric scale.
4. Change each student's weight (mass) from kg to cm^3.

Example:

50 kg 50 x 1 000 = 5 000 g or 5 000 cm^3

Because body weight is 97% water, this activity uses the approximate equivalents of 1 cm^3 to 1 g of body weight.

Worksheet

The Cubic Meter

Construct a cubic meter by assembling twelve meter sticks into a cube.

How well do you think that our class can fit inside? Explain.

Find your weight/mass in kilograms on the metric scale.
Change your weight/mass to cubic centimeters.
List this information on a chart.

Answer the following questions:

1. What is a gram? _____
2. What is a cubic centimeter? _____
3. How many centimeters are there in a meter? _____
4. How many square centimeters are there in a square meter? _____
5. How many cubic centimeters are there in a cubic meter? _____

Answers:

1. A basic unit of weight in the metric system is the gram.
2. The gram is equal, almost exactly to the weight of one cubic centimeter of water. A cubic centimeter is such that its length equals one centimeter, its width equals one centimeter, and its height equals one centimeter.
3. 100 cm = 1 meter
4. A = 100 cm x 100 cm
   A = 10 000 cm^2, so therefore, 10 000 cm^2 = 1 m^2
5. The volume of a cubic meter is 100cm x 100cm x 100cm or 1 000 000 cm^3

RESOURCES

Academic Industries Inc.
P.O. Box 428
Baychester Station
Bronx, NY 10469

Activity Resources Co. Inc.
P.O. Box 4875
Hayward, CA 94540

American Guidance Service
Circle Pines, MI 55014

Creative Publications
5005 West 110th Street
Oak Lawn, IL 60453

Cuisenaire Co. of America, Inc.
12 Church Street, Box D
New Rochelle, NY 10802

Dale Seymour Publications
P. O. Box 10888
Palo Alto, CA 94303

Delta Education
Box M
Mathematics Department
Nashua, NH 03061

Delta X Industries, Inc.
Deptartment 53CD
P. O. Box 442
Mariette, WI 54143

Didax Educational Resources, Inc.
6 Doulton Place
Peabody, MA 01960

DLM Teaching Resources
P. O. Box 4000
One DLM Park
Allen, TX 75002

Educational Teaching Aids
199 Carpenter Avenue
Wheeling, IL 60090

Gamco Industries, Inc.
Box 1911 Q
Big Spring, TX 79720

La Pine Scientific Company
Department A432
6003 South Knox Avenue
Chicago, IL 60629

NASCO
901 Janesville Avenue
Fort Atkinson, WI 53538